Measuring regional and global integration

Foundations of structural gravity

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Module 2 — Part 5

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Market structures for Armington-Anderson-type gravity

Market structures

1. Perfect competition
2. Endowment economy
3. Monopolistic competition

Case 1 - Perfect competition

- perfect competition: price of good is simply marginal cost
- each worker produces A_i units and costs w_i
- marginal cost equal to factory gate price: $p_i = \frac{w_i}{A_i}$
- hence including trade costs $p_{ij} = \tau_{ij}\frac{w_i}{A_i}$
  $\rightarrow$ rearranging yields no-arbitrage condition: $\frac{p_{ij}}{p_i} = \tau_{ij}$

Case 1 - Perfect competition

- Substituting price into the demand equation yields

$$
\begin{aligned}
X_{ij} &= a_{ij} \left(\tau_{ij} \frac{w_i}{A_i} \right)^{1-\sigma} Y_j P_j^{\sigma-1} \\
&= \left(\frac{w_i}{A_i} \right)^{1-\sigma} Y_j P_j^{\sigma-1} a_{ij} \tau_{ij}^{1-\sigma}
\end{aligned}
$$

$\rightarrow$ general gravity!

Case 2 - Endowment economy

- Assume now that country is endowed with fixed amount of the good, M_i
- Country allocates it across consumers in all countries to maximize profits
- Optimization problem is the

$$\max_{\{q_{ij}\}_{j\in S}} \sum_{j\in S} p_{ij} q_{ij} \quad \text{s.t.} \quad \sum_j \tau_{ij} q_{ij} \leq M_i \quad \text{and} \quad q_{ij} = a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1}$$

Case 2 - Endowment economy

- Substitute second constraint into maximand and first constraint

$$\max_{\{q_{ij}\}_{j\in S}} \sum_{j\in S} p_{ij} a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1} \quad \text{s.t.} \quad \sum_j \tau_{ij} a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1} \leq M_i$$

- FOC wrt p_{ij} yield

$$(1-\sigma) a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1} = -\lambda \tau_{ij} \sigma p_{ij}^{-\sigma-1} Y_j P_j^{\sigma-1}$$
$$\Leftrightarrow p_{ij} = \frac{\sigma}{\sigma-1} \lambda \tau_{ij}$$

→ same *net* price is charged in all destinations

Case 2 - Endowment economy

- Substituting in second FOC

$$
\begin{aligned}
M_i &= \sum_j \left(\frac{\sigma}{\sigma-1}\lambda\tau_{ij}\right)^{-\sigma}\tau_{ij}a_{ij}Y_jP_j^{\sigma-1} \\
\Leftrightarrow \left(\frac{\sigma}{\sigma-1}\lambda\right) &= \left(\frac{\sum_j \tau_{ij}^{1-\sigma}a_{ij}Y_jP_j^{\sigma-1}}{M_i}\right)^{\frac{1}{\sigma}} \\
\Leftrightarrow p_{ij} &= \left(\frac{\sum_j \tau_{ij}^{1-\sigma}a_{ij}Y_jP_j^{\sigma-1}}{M_i}\right)^{\frac{1}{\sigma}}\tau_{ij}
\end{aligned}
$$

Case 2 - Endowment economy

- Substituting back into demand equation

$$X_{ij} = a_{ij} \left(\left(\frac{\sum_j^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1}}{M_i} \right)^{\frac{1}{\sigma}} \tau_{ij} \right)^{1-\sigma} Y_j P_j^{\sigma-1}$$

$$= M_i^{\frac{\sigma-1}{\sigma}} \left(\sum_j \tau_{ij}^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1} \right)^{\frac{1-\sigma}{\sigma}} Y_j P_j^{\sigma-1} \tau_{ij}^{1-\sigma} a_{ij}$$

- This looks already almost like a structural gravity equation!

Case 2 - Endowment economy

- Suppose $\tau_{ii} = 1$
- As price net of transportation costs is equal across all locations, we have

$$
\begin{aligned}
Y_i &= M_i p_{ii} \\
&= M_i \left(\frac{\sum_j \tau_{ij}^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1}}{M_i} \right)^{\frac{1}{\sigma}} \tau_{ii} \\
&= M_i^{\frac{\sigma-1}{\sigma}} \left(\sum_j \tau_{ij}^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1} \right)^{\frac{1}{\sigma}}
\end{aligned}
$$

Case 2 - Endowment economy

- Substituting back in yields

$$X_{ij} = \frac{Y_i}{\Pi_i^{1-\sigma}} \frac{Y_j}{P_j^{1-\sigma}} a_{ij} \tau_{ij}^{1-\sigma} \quad \text{with} \quad \Pi_i = \left(\sum_j Y_j P_j^{\sigma-1} a_{ij} \tau_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

→ which is a structural gravity equation!