Supply-Side Input-Output Models

ADB-CI Virtual Workshop on Input-Output Analysis

Outline

- Recap of Demand-side Input-Output Models
    - Leontief Quantity Model
    - Leontief Price Model
- Supply-side Quantity Model, Ghosh
    - Examples
- Re-interpretation as a price model
    - Examples

Input-Output Model

	Sector 1	Sector 2	Sector 3	Final Demand	Total
Sector 1					
Sector 2		**Z**		**f**	**x**
Sector 3					
Primary input		**v’**			
Total		**x’**			

- **Z** is the intermediate consumption matrix
- **f** is the final demand vector
- **x** is the vector of total output
- **v'** is the vector of value added
- **x’** is the vector of total input

Recap: Demand-driven models

- Demand-pull Input-Output (IO) Quantity Model

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f}$$

$$\mathbf{x} = \mathbf{Lf}$$

where $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1};\ \mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$

	Sector 1	Sector 2	Sector 3	Final Demand	Total
Sector 1					
Sector 2		$\mathbf{Z}$		$\mathbf{f}$	$\mathbf{x}$
Sector 3					
Primary input					
Total					

Recap: Demand-driven models

- Cost-push IO Price Model

$$\mathbf{x}' = \mathbf{i}'\mathbf{Z} + \mathbf{v}'$$

$$\tilde{\mathbf{p}} = \mathbf{L}'\mathbf{v}_c$$

$$\text{where } \mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1};\ \mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}; \mathbf{v}_c = \mathbf{v}'\hat{\mathbf{x}}^{-1}$$

	Sector 1	Sector 2	Sector 3	Final Demand	Total
Sector 1					
Sector 2		$\mathbf{Z}$			
Sector 3					
Primary input		$\mathbf{v}'$			
Total		$\mathbf{x}'$			

Quantity Models

Demand-driven quantity model

- Production is a function of final demand, given input coefficients (i.e., production technology).
- Relates output and product leaving the production process
- Equation form:

$$\mathbf{x} = \mathbf{Lf}$$

Supply-driven quantity model (Ghosh, 1958)

- Production is determined by value added
- Relates output and value entering the production process
- Producers must induce sales in order to achieve a desired level of income.
- Equation form:

$$\mathbf{x} = \mathbf{G}'\mathbf{v} \quad (4.1)$$

Supply-side Quantity Model, Ghosh (1958)

- The supply-side IO model is made operational by "rotating" our vertical (column) view from demand-side model to a horizontal (row) one as in the cost-push I-O price model: $\mathbf{x}' = \mathbf{i}'\mathbf{Z} + \mathbf{v}'$.

Demand-side vs Supply-side IO models

	Sector 1	Sector 2	Sector 3	Final Demand	Total
Sector 1					x_1
Sector 2					
Sector 3					
Primary input					
Total	x_1				

In the demand-driven model, we derive the **technical coefficients matrix $\mathbf{A}$** by dividing each column of $\mathbf{Z}$ with their corresponding total output $\mathbf{x}$.

In the supply-driven model, we derive a **direct-output coefficients matrix $\mathbf{B}$** by dividing each row of $\mathbf{Z}$ with their corresponding total output $\mathbf{x}$.

Quantity Models

Demand-driven quantity model

- Production is a function of final demand, given input coefficients (i.e., production technology).
- Relates output and product leaving the production process

Equation form:

$$\mathbf{x} = \mathbf{Lf}$$

- Where $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$

Supply-driven quantity model (Ghosh, 1958)

- Production is determined by value added
- Relates output and value entering the production process

Equation form:

$$\mathbf{x} = \mathbf{G}'\mathbf{v} \quad (4.1)$$

- Where $\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1}$ (4.2)

Supply-side Quantity Model, Ghosh (1958)

- Derive B by dividing each row of Z by the gross output of the sector associated with that row.
- For a two-sector economy:

Given: $\mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \end{bmatrix}$; $\hat{\mathbf{x}} = \begin{bmatrix} \mathrm{x}_1 & 0 \\ 0 & \mathrm{x}_2 \end{bmatrix}$

Find: $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \rightarrow \mathbf{B} = \begin{bmatrix} & \\ & \end{bmatrix}$

$$\mathbf{B} = \hat{\mathbf{x}}^{-1}\mathbf{Z} \qquad (4.3)$$

- Where each b_{ij} in B is called an **allocation coefficient**, which represents the **distribution of sector *i*'s outputs across sectors *j*** (including itself).

Supply-side Quantity Model, Ghosh (1958)

- The relationship between output and value-added, summarized in (4.1) as $\mathbf{x} = \mathbf{G}'\mathbf{v}$ can also be showed in terms of changes, or

$$\Delta\mathbf{x} = \mathbf{G}'\Delta\mathbf{v} \qquad (4.4)$$

Note that the basic assumption of the supply-side model is that **output distributions are stable in an economic system**.

  - This means that each output coefficient b_{ij} is fixed, which makes each g_{ij} fixed as well.

Interpreting the output inverse matrix $\mathbf{G}$

- From (4.1), $\mathbf{x} = \mathbf{G}'\mathbf{v}$

$$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix}' \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} g_{11} & \cdots & g_{n1} \\ \vdots & \ddots & \vdots \\ g_{1n} & \cdots & g_{nn} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$$

- If we are to get a slice of one equation from above, say sector 1:

$$x_1 = v_1 g_{11} + \cdots + v_i g_{i1} + \cdots + v_n g_{n1}$$

- Or generally, say sector j:

$$x_j = v_1 g_{1j} + \cdots + v_i g_{ij} + \cdots + v_n g_{nj}$$

Interpreting the output inverse matrix $\mathbf{G}$

- Let's take the sector 1 equation:

$$x_1 = v_1 g_{11} + \cdots + v_i g_{i1} + \cdots + v_n g_{n1}$$

- And express it in terms of changes.

$$\Delta x_1 = \Delta v_1 g_{11} + \cdots + \Delta v_i g_{i1} + \cdots + \Delta v_n g_{n1}$$

- Question: Suppose there is a $1 change in the availability of primary inputs to sector 1. In other words, suppose $\Delta v_1 = 1$, while other primary inputs remain constant. What is the change in x_1 (or Δx_1)?
- Answer: Δx_1 is measured by g_{11}.

Interpreting the output inverse matrix $\mathbf{G}$

$$\mathbf{G} = \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix}$$

- Therefore, each g_{ij} measures the effect **on sector** j output a $1 change in primary inputs to sector i.
- In comparison to demand-driven models where $\Delta\mathbf{f}$ are considered as exogenous demand changes, $\Delta\mathbf{v}$ represent exogenous supply changes.

Recall: Demand-side Output Multipliers

- Recall: A **simple output multiplier** for sector j, $m(o)_j$, is defined as the total value of production in **all** sectors to satisfy an additional $1 worth of final demand for **sector *j***'s output.

$$\mathbf{m}(o) = \mathbf{i}'\mathbf{L}$$

$$\mathbf{m}(o) = \begin{bmatrix} 1 & \cdots & 1_n \end{bmatrix} \begin{bmatrix} l_{11} & \cdots & l_{1n} \\ \cdots & \ddots & \vdots \\ l_{n1} & \cdots & l_{nn} \end{bmatrix} = \begin{bmatrix} m(o)_1 & \cdots & m(o)_j & \cdots & m(o)_n \end{bmatrix}$$

  - That is, the vector of simple output multipliers is merely comprised of the **column sum** of the Leontief inverse $\mathbf{L}$.

Supply-side Output Multipliers

- Define: **Input (or supply) multiplier** represents the total output through **all** sectors of the economy that would be associated with a $1 change in primary inputs for **sector** $\boldsymbol{i}$.

$$\mathbf{m}(s) = \mathbf{Gi} \quad (4.5)$$

$$\mathbf{m}(s) = \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1_n \end{bmatrix} = \begin{bmatrix} m(s)_1 \\ \vdots \\ m(s)_i \\ \vdots \\ m(s)_n \end{bmatrix}$$

- That is, the vector of input multipliers is merely comprised of the **row sum** of the output inverse **G**.
- $\mathbf{m}(s)$ is the supply-side analog of the simple output multiplier $\mathbf{m}(o)$

Example

Ghosh quantity model

Supply-side Output Multipliers

- Policy implications of multipliers include:
  - Assessing where an additional dollar’s worth of provision of primary resource would be most beneficial to the total economy.
  - Assessing potential contracting effects of shortages in primary inputs to a particular sector.

Criticisms to the Ghosh Quantity Model

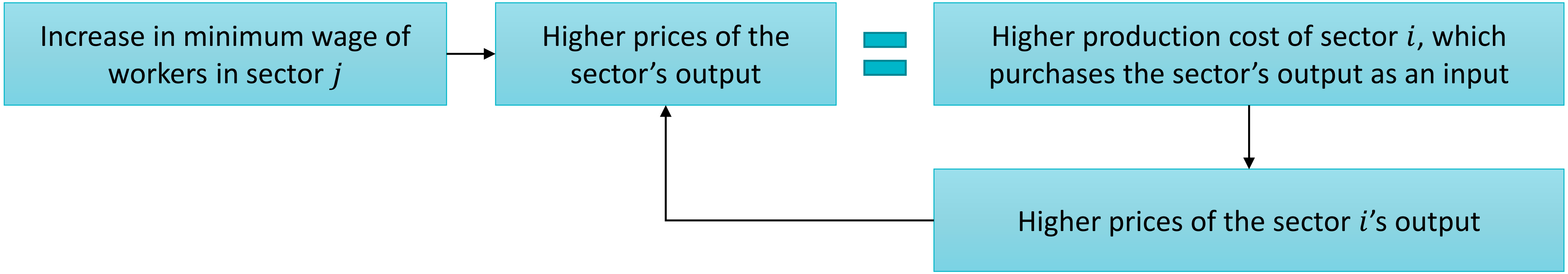
- The Ghosh model may only be applicable in the context of an economy experiencing severe excess demand, with government-imposed restrictions on supply patterns, such that supply distribution patterns remain constant. This is not a good representation of the modern world.
- Primary input increases in sector j are transmitted forward as output increases in all sectors that buy from j, without any corresponding increases in primary input use of those sectors. This could not exist simultaneously with the Leontief model which assumes that inputs are used in fixed proportions.

Supply-side Price Model, Dietzenbacher (1997)

- Dietzenbacher (1997) proposed to view the Ghosh model as a price model instead of a quantity model to overcome its criticisms.
- We refer to this alternative view as the Ghosh Price Model.
- Under the Ghosh Price Model:
    - Elements of the supply-driven model are **viewed as values** instead of quantities.
    - As opposed to the quantity model, which holds prices as fixed, the **Ghosh Price Model assumes that all quantities are fixed**.
    - Therefore, this model can also be considered a cost-push IO model

Supply-side Price Model, Dietzenbacher (1997)

- Changes in the costs of primary inputs of sector j are **transmitted throughout the economy** as they are completely passed on by producers in the prices of their products that are purchased by other intermediate users, who in turn also increase their prices accordingly, and so on.



Supply-side Price Model, Dietzenbacher (1997)

- We can identify the relative price changes as the ratios of the elements in $\mathbf{x_0}$ (i.e., base quantities) to those in $\mathbf{x_1}$ (i.e., new quantities). Define:

$$\boldsymbol{\pi} = (\widehat{\mathbf{x_0}})^{-1}\mathbf{x_1} \qquad (4.6)$$

$$\boldsymbol{\pi} = \begin{bmatrix} 1/x_{0_1} & \cdots & 0 \\ \cdots & \ddots & \vdots \\ 0 & \cdots & 1/x_{0_n} \end{bmatrix} \begin{bmatrix} x_{1_1} \\ \vdots \\ x_{1_n} \end{bmatrix}$$

$$\boldsymbol{\pi} = \begin{bmatrix} \pi_1 \\ \vdots \\ \pi_n \end{bmatrix}$$

- Each π_i refers to the relative price change in the output of sector i as a result of the exogenous change in price of primary inputs.

Examples

Ghosh price model