Output Multiplier decomposition

ADB-CI Virtual Workshop on Input-Output Analysis

Outline

- Why decompose
- Multiplier decomposition for a single region
- Multiplier decomposition for multiple regions
- Stone's decomposition

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Why decompose multipliers?

- output multipliers tell us the increase in output for an exogenous change in a sector's final demand.
  - can we identify where the increase in output is happening?

- can we decompose growth into its constituent parts?

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Single region

Single-region context

$$x = Ax + f \tag{1}$$

Gross output = Intermediate inputs + final demand

Define

$$\tilde{A} = \begin{bmatrix} a_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & a_{NN} \end{bmatrix}$$

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Single-region context

$$x = Ax + f \tag{1}$$

$$x = Ax - \tilde{A}x + \tilde{A}x + f$$
$$x = \tilde{A}x + \left(A - \tilde{A}\right)x + f$$
$$x = \left(I - \tilde{A}\right)^{-1}\left(A - \tilde{A}\right)x + \left(I - \tilde{A}\right)^{-1}f$$

Define $A^* = \left(I - \tilde{A}\right)^{-1}\left(A - \tilde{A}\right)$

$$x = A^*x + \left(I - \tilde{A}\right)^{-1}f \tag{2}$$

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Single-region context

$$x = A^{*}x + \left(I - \tilde{A}\right)^{-1} f \qquad (2)$$

To get an expression for $A^{*}x$, pre-multiply A^{*} to both sides of (2)

$$A^{*}x = A^{*2}x + A^{*}\left(I - \tilde{A}\right)^{-1} f$$

Plugging to (2)

$$x = A^{*2}x + (I + A^{*})\left(I - \tilde{A}\right)^{-1} f$$

$$x = (I - A^{*2})^{-1}(I + A^{*})\left(I - \tilde{A}\right)^{-1} f \qquad (3)$$

$x = Bf$

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Single-region context

$$x = (I - A^{*2})^{-1}(I + A^{*})\left(I - \tilde{A}\right)^{-1} f \qquad (3)$$

M_3 M_2 M_1

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Single-region context

$$x = (I - A^{*2})^{-1}(I + A^{*})\left(I - \tilde{A}\right)^{-1} f \qquad (3)$$

M_3 M_2 M_1

M_1 = transfer effect

intrasectoral effect of increase in final demand

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Single-region context

$$x = (I - A^{*2})^{-1}(I + A^{*})\left(I - \tilde{A}\right)^{-1} f \qquad (3)$$

M_3 M_2 M_1

M_1 = transfer effect

intrasectoral effect of increase in final demand

M_2 = open loop effect

increase in one sector will spillover in other sectors (but feedback effects not captured)

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Single-region context

$$x = (I - A^{*2})^{-1}(I + A^{*})\left(I - \tilde{A}\right)^{-1} f \qquad (3)$$

M_3 M_2 M_1

M_1 = transfer effect — intrasectoral effect of increase in final demand

M_2 = open loop effect — increase in one sector will spillover in other sectors (but feedback effects not captured)

M_3 = closed loop effect — increases in other sectors affect the original sector, which in turn generates further spillover and so on

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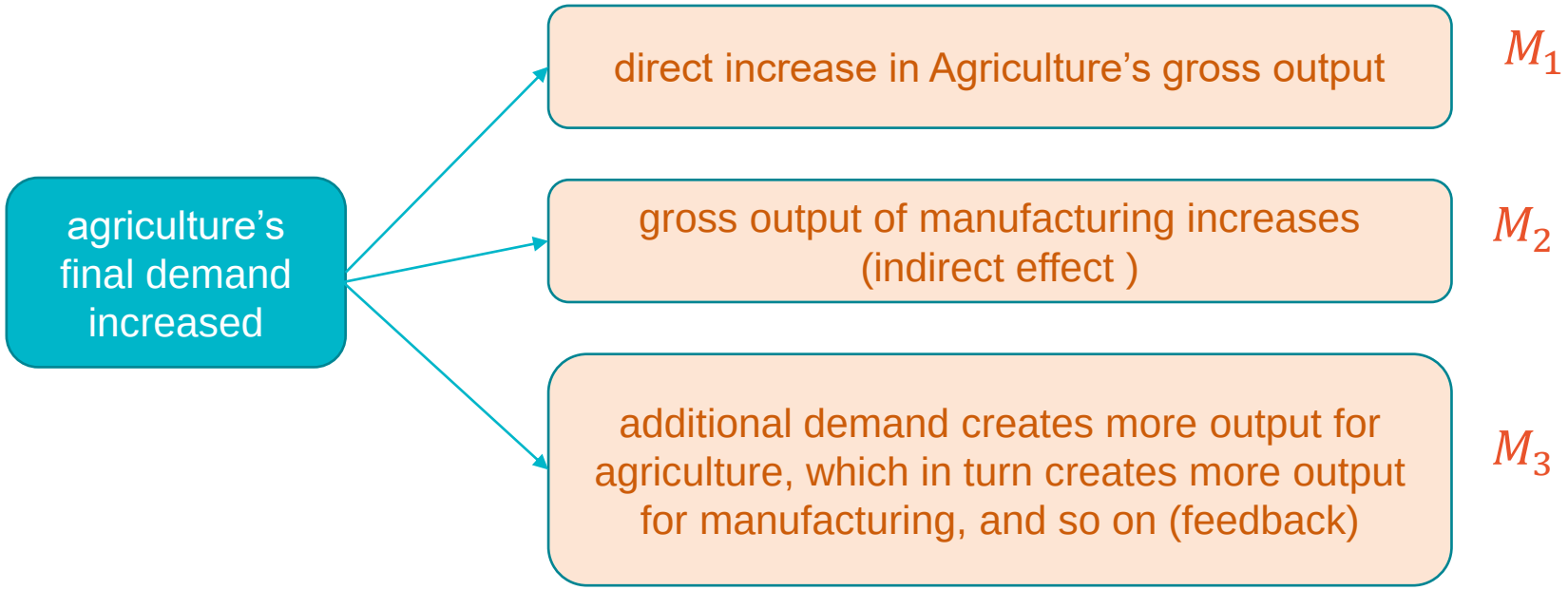
Single-region context

$$x = (I - A^{*2})^{-1}(I + A^{*})\left(I - \tilde{A}\right)^{-1} f \qquad (3)$$

Hypothetical country A with 2 sectors: Agriculture and Manufacturing



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Examples

Multiregional

Multiregional context

$$x = Ax + f \tag{1}$$

Gross output = Intermediate inputs + final demand

Define

$$\tilde{A} = \begin{bmatrix} A^{ss} & 0 \\ 0 & A^{rr} \end{bmatrix}$$

where A^{ss} is a blocked diagonal matrix for country

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Multiregional context

$$x = Ax + f$$

$$M_1 = \left(I - \tilde{A}\right)^{-1} = \begin{bmatrix} (I - A^{ss})^{-1} & 0 \\ 0 & (I - A^{rr})^{-1} \end{bmatrix} \quad (4)$$

$$M_2 = (I + A^*) = \begin{bmatrix} I & (I - A^{rr})^{-1} A^{rs} \\ (I - A^{ss})^{-1} A^{sr} & I \end{bmatrix} \quad (5)$$

where $A^* = \left(I - \tilde{A}\right)^{-1}\left(A - \tilde{A}\right)$

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Multiregional context

$$x = Ax + f$$

$$M_1 = \left(I - \tilde{A}\right)^{-1} = \begin{bmatrix} (I - A^{ss})^{-1} & 0 \\ 0 & (I - A^{rr})^{-1} \end{bmatrix} \quad (4)$$

$$M_2 = (I + A^*) = \begin{bmatrix} I & (I - A^{rr})^{-1} A^{rs} \\ (I - A^{ss})^{-1} A^{sr} & I \end{bmatrix} \quad (5)$$

where $A^* = \left(I - \tilde{A}\right)^{-1}\left(A - \tilde{A}\right)$

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Multiregional context

$$x = Ax + f$$

And by using matrix multiplication to obtain A^{*2}

$$M_3 = (I - A^{*2})^{-1}$$
$$M_3 = \begin{bmatrix} (I - (I - A^{rr})^{-1} A^{rs} (I - A^{ss})^{-1} A^{sr})^{-1} & 0 \\ 0 & (I - (I - A^{ss})^{-1} A^{sr} (I - A^{rr})^{-1} A^{rs})^{-1} \end{bmatrix} \quad (5)$$

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Multiregional context

intraregional effect

$$M_1 = \left(I - \tilde{A}\right)^{-1} = \begin{bmatrix} (I - A^{ss})^{-1} & 0 \\ 0 & (I - A^{rr})^{-1} \end{bmatrix}$$

interregional spillover effect

$$M_2 = (I + A^*) = \begin{bmatrix} I & (I - A^{rr})^{-1} A^{rs} \\ (I - A^{ss})^{-1} A^{sr} & I \end{bmatrix}$$

interregional feedback effect

$$M_3 = \begin{bmatrix} (I - (I - A^{rr})^{-1} A^{rs} (I - A^{ss})^{-1} A^{sr})^{-1} & 0 \\ 0 & (I - (I - A^{ss})^{-1} A^{sr} (I - A^{rr})^{-1} A^{rs})^{-1} \end{bmatrix}$$

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Stone's decomposition

Stone provides a way to isolate net effects

$$x = If + \underbrace{(M_1 - I)f}_{\widetilde{M}_1} + \underbrace{(M_2 - I)M_1 f}_{\widetilde{M}_2} + \underbrace{(M_3 - I)M_2 M_1 f}_{\widetilde{M}_3}$$

direct effects	net intraregional effects	net interregional spillover effects	net interregional feedback effects

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Examples

Thank you.