Input-Output Models at the Regional Level

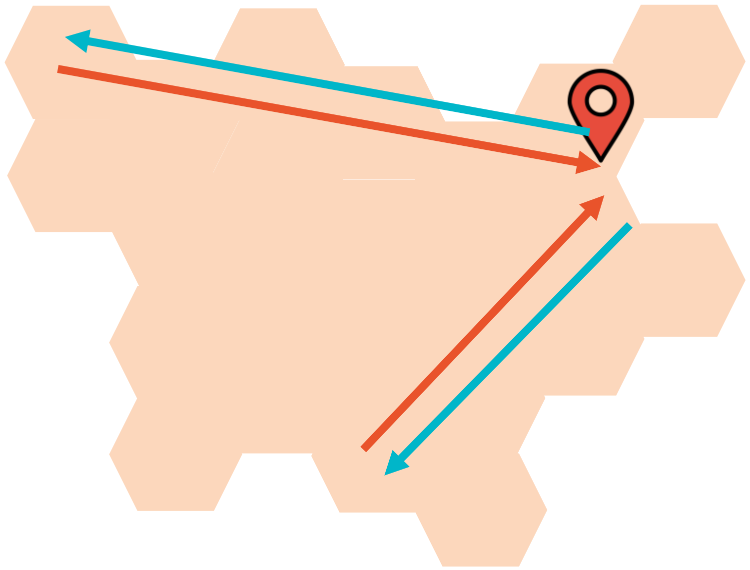
Julian Thomas Alvarez
John Arvin Bernabe
Consultants
Asian Development Bank

Features of the regional economy:

1 Unique production

2 Use-dependence

3 Supply-dependence



National Economic Territory

Objectives and outline

To demonstrate the versatility of input-output analysis with regionalized tables and their uses for analyzing production linkages and spillovers between any two region and sector pairs.

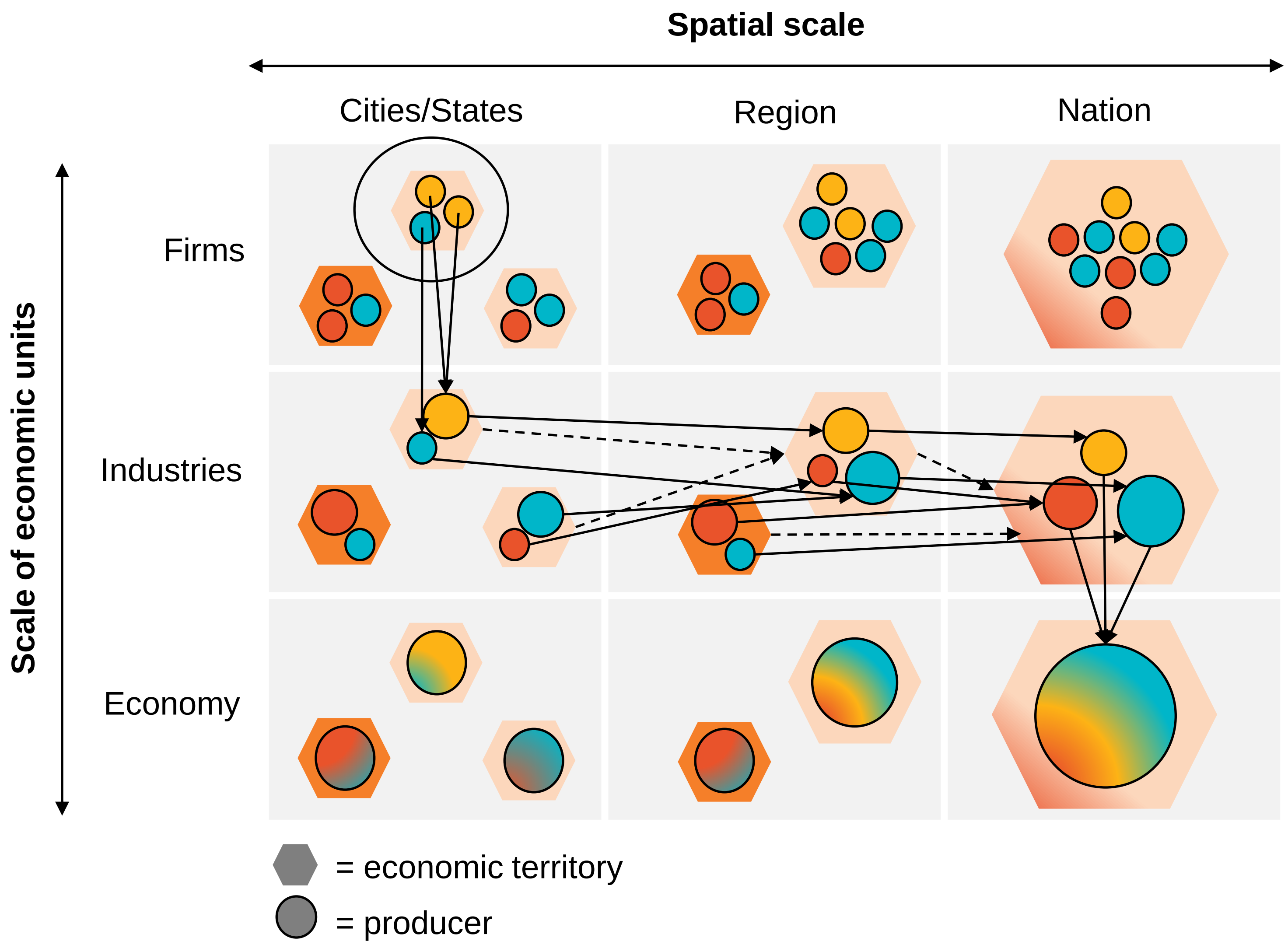
Outline:

I. Single region models

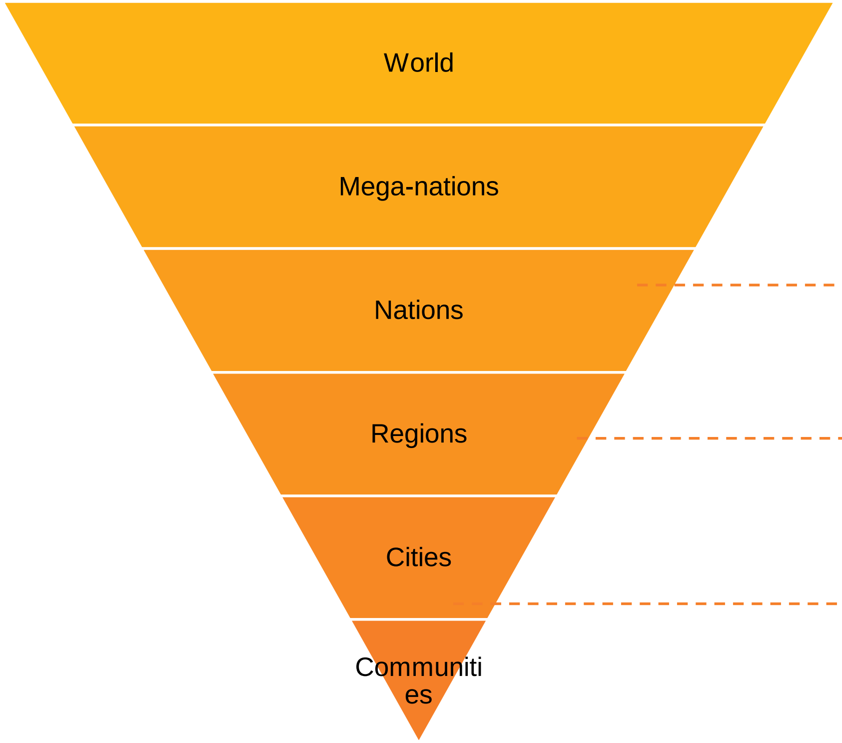
II. Two or many region models
   A. Interregional models (“Isard approach”)
   B. Multiregional models (“Chenery-Moses approach”)

III. Hands-on exercises

Scaling assumptions in IOTs



Scaling assumptions in IOTs



Main types of input-output models at the regional level

Single-region

National input-output tables
Single region ≈ 1 national economy
e.g., Philippines

Subnational input-output tables
Single region ≈ 1 local region
e.g., Metro Manila

City input-output tables
Single region ≈ 1 city
e.g., Manila city

Regionalization

Regionalization

Two or many-region

International input-output tables
Several regions ≈ several national economies
e.g., Philippines, Indonesia, and rest of the world

National input-output tables
Several regions ≈ local regions
e.g., Metro Manila and rest of the Philippines

Subnational input-output tables
Several regions ≈ cities
e.g., Manila and Mandaluyong cities

"Regionalizing" input-output tables

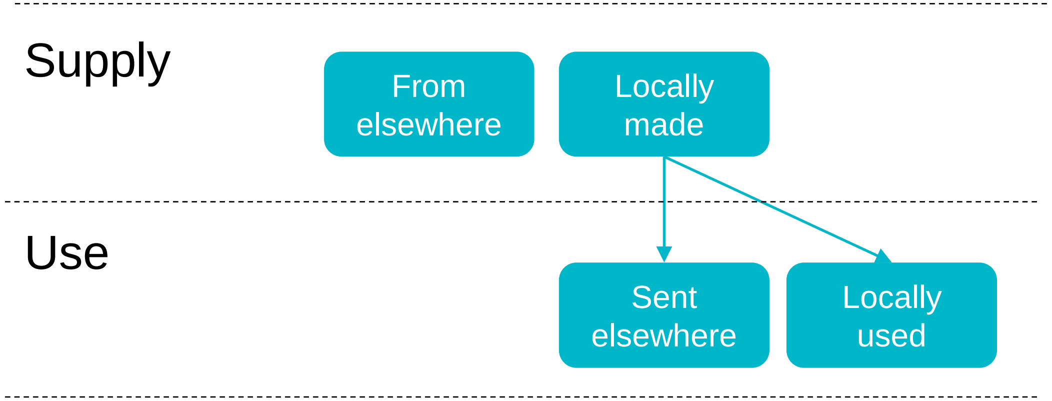
Assume that the national economy has two regions *r* and *s*.

$$\begin{pmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} \end{pmatrix} = \mathbf{A} = \begin{pmatrix} \boldsymbol{A}^{rs} & \\ & \end{pmatrix} = \begin{matrix} \begin{pmatrix} a_{11}^{rr} & \cdots & a_{1j}^{rr} \\ \vdots & \ddots & \vdots \\ a_{i1}^{rr} & \cdots & a_{ij}^{rr} \end{pmatrix} \begin{pmatrix} a_{11}^{rs} & \cdots & a_{1j}^{rs} \\ \vdots & \ddots & \vdots \\ a_{i1}^{rs} & \cdots & a_{ij}^{rs} \end{pmatrix} \\ \begin{pmatrix} a_{11}^{sr} & \cdots & a_{1j}^{sr} \\ \vdots & \ddots & \vdots \\ a_{i1}^{sr} & \cdots & a_{ij}^{sr} \end{pmatrix} \begin{pmatrix} a_{11}^{ss} & \cdots & a_{1j}^{ss} \\ \vdots & \ddots & \vdots \\ a_{i1}^{ss} & \cdots & a_{ij}^{ss} \end{pmatrix} \end{matrix}$$

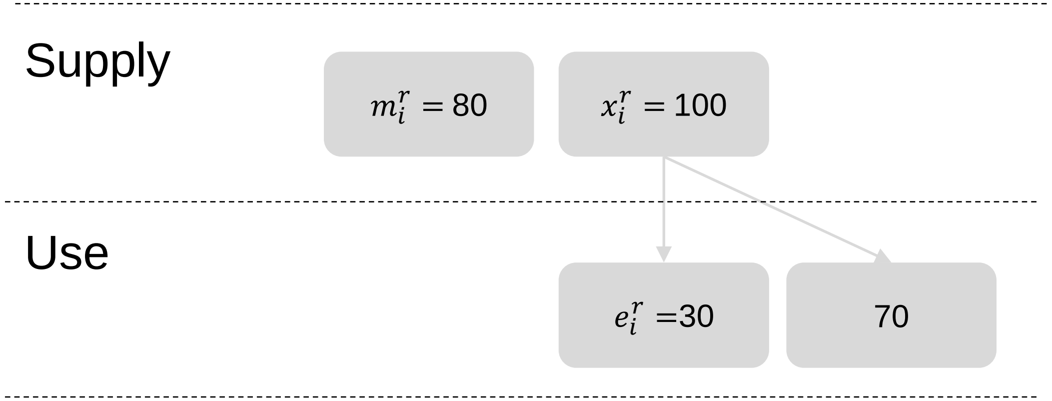
How do you obtain region-specific input coefficients?
One approach is to use **regional supply percentages**.

Regional supply percentages, $\mathbf{p}^r$

An estimate of the percentage of product i available in r that was produced there.



Example: i = Wheat products



Regional supply percentage p_i^r of sector *i* in region *r* is given by:

$$p_i^r = \frac{(x_i^r - e_i^r)}{(x_i^r - e_i^r + m_i^r)} = \frac{\text{Locally made for local consumption}}{\text{Locally available products}}$$

$$p_i^r = \frac{100 - 30}{100 - 30 + 80} = 46.67\%$$

Single-region model using $\mathbf{p}^r$

$$\mathbf{x}^r = \hat{\mathbf{p}}^r \mathbf{A}\mathbf{x}^r + \mathbf{f}^r$$

$$\mathbf{x}^r - \hat{\mathbf{p}}^r \mathbf{A}\mathbf{x}^r = \mathbf{f}^r$$

$$\mathbf{x}^r(\mathbf{I} - \hat{\mathbf{p}}^r \mathbf{A}) = \mathbf{f}^r$$

$$\underbrace{\mathbf{x}^r}_{\text{Total output in region } r} = \underbrace{(\mathbf{I} - \hat{\mathbf{p}}^r \mathbf{A})^{-1}}_{\text{Total intraregional production requirements in } r} \underbrace{\mathbf{f}^r}_{\text{Exogenous final demand for products in region } r}$$

$\mathbf{x}^r$ = vector of gross outputs in *r*

$\hat{\mathbf{p}}^r$ = regional supply percentages in r

$\mathbf{A}$ = Input coefficients ("unregionalized" i.e., national economy)

$\mathbf{f}^r$ = final demand for *r* products

Basic question:

How much production in region r is required to satisfy final demands in r?

Single-region model using $\mathbf{p}^r$

Numerical example:

- Interested in region *r* in the national economy
- 2 sectors in the economy and respective regions (n=2)
- National IOT is available; regional IOT is not available
- Are the following information enough to build a model?

For the national economy:

Sector	Intermediate consumption		Output
	1	2	
1	310	400	1,200
2	280	220	900

For the regional economy r:

Sector	Output	Exports	Imports
1	300	120	200
2	200	30	120

Single-region model using $\mathbf{p}^r$

For the national economy:

Sector	Intermediate consumption		Output
	1	2	
1	310	400	1,200
2	280	220	900

For the regional economy r:

Sector	Output	Exports	Imports
1	300	120	200
2	200	30	120

Find $\mathbf{A}$.

$$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$$

$$\mathbf{A} = \begin{pmatrix} 310/1200 & 400/900 \\ 280/1200 & 220/900 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{0.2583} & \mathbf{0.4444} \\ \mathbf{0.2333} & \mathbf{0.2444} \end{pmatrix}$$

Find $\hat{\mathbf{p}}^r$.

$$p_i^r = \frac{(x_i^r - e_i^r)}{(x_i^r - e_i^r + m_i^r)}$$

$$\mathbf{p}^r = \begin{bmatrix} (300 - 120)/(300 - 120 + 200) \\ (200 - 30)/(200 - 30 + 120) \end{bmatrix}$$

$$\mathbf{p}^r = \begin{pmatrix} 0.4737 \\ 0.5862 \end{pmatrix}$$

$$\hat{\mathbf{p}}^r = \begin{pmatrix} \mathbf{0.4737} & \mathbf{0} \\ \mathbf{0} & \mathbf{0.5862} \end{pmatrix}$$

Single-region model using $\mathbf{p}^r$

$$\hat{\mathbf{p}}^r = \begin{pmatrix} 0.4737 & 0 \\ 0 & 0.5862 \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} 0.2583 & 0.4444 \\ 0.2333 & 0.2444 \end{pmatrix}$$

Find $\mathbf{A}^{\mathbf{rr}}$

$$\mathbf{A}^{\mathbf{rr}} = \hat{\mathbf{p}}^r \mathbf{A}$$

$$\mathbf{A}^{\mathbf{rr}} = \begin{pmatrix} .122 & .211 \\ .137 & .143 \end{pmatrix}$$

Find $\mathbf{f}^r$ using $\mathbf{A}^{\mathbf{rr}}$ and $\mathbf{x}^r$

$$\boldsymbol{Z}^{\boldsymbol{rr}} = \mathbf{A}^{\mathbf{rr}} \hat{\mathbf{x}}^r$$

$$\boldsymbol{Z}^{\boldsymbol{rr}} = \begin{pmatrix} .122 & .211 \\ .137 & .143 \end{pmatrix} \begin{pmatrix} 300 & 0 \\ 0 & 200 \end{pmatrix} = \begin{pmatrix} 36.7 & 42.1 \\ 41.0 & 28.7 \end{pmatrix}$$

$$\mathbf{f}^r = \mathbf{x}^r - \mathbf{Z}^{\mathbf{rr}} \mathbf{i}$$

$$\mathbf{f}^r = \begin{pmatrix} 300 \\ 200 \end{pmatrix} - \begin{pmatrix} 78.8 \\ 69.7 \end{pmatrix} = \begin{pmatrix} 221.2 \\ 130.3 \end{pmatrix}$$

Single-region model using $\mathbf{p}^r$

Setting up the model:

$$\mathbf{A^{rr}} = \begin{pmatrix} .122 & .211 \\ .137 & .143 \end{pmatrix} \qquad \mathbf{f}^r = \begin{pmatrix} 221.2 \\ 130.3 \end{pmatrix}$$

$$\mathbf{I} - \mathbf{A^{rr}} = \begin{pmatrix} 1 & 0 \\ 0 & .1 \end{pmatrix} - \begin{pmatrix} .122 & .211 \\ .137 & .143 \end{pmatrix}$$

$$\mathbf{I} - \mathbf{A^{rr}} = \begin{pmatrix} 0.878 & -0.211 \\ -0.137 & .857 \end{pmatrix}$$

$$(\mathbf{I} - \mathbf{A^{rr}})^{-1} = \begin{pmatrix} 1.185 & 0.291 \\ 0.189 & 1.214 \end{pmatrix}$$

$$(\mathbf{I} - \mathbf{A^{rr}})^{-1}\mathbf{f}^r = \begin{pmatrix} 1.185 & 0.291 \\ 0.189 & 1.214 \end{pmatrix} \begin{pmatrix} 221.2 \\ 130.3 \end{pmatrix}$$

$$(\mathbf{I} - \mathbf{A^{rr}})^{-1}\mathbf{f}^r = \begin{pmatrix} 300 \\ 200 \end{pmatrix} = \mathbf{x}^r$$

which yields the same levels of output as given.

Only evaluate the final demand for sector 1 in r:

$$(\mathbf{I} - \mathbf{A^{rr}})^{-1}\mathbf{f}^{r*} = \begin{pmatrix} \boxed{1.185} & 0.291 \\ \boxed{0.189} & 1.214 \end{pmatrix} \begin{pmatrix} 100 \\ 0 \end{pmatrix}$$

$$(\mathbf{I} - \mathbf{A^{rr}})^{-1}\mathbf{f}^{r*} = \begin{pmatrix} 118.5 \\ 18.9 \end{pmatrix}$$

$$\mathbf{L^{rr}} = (\mathbf{I} - \mathbf{A^{rr}})^{-1} = [l_{ij}^{rr}]$$

Each element $[l_{ij}^{rr}]$ refers to the total production required from sector i in region r per unit of demand for sector j output in the same region r.

Missing:
Impacts from/to other regions in the national economy

SECTION SUMMARY

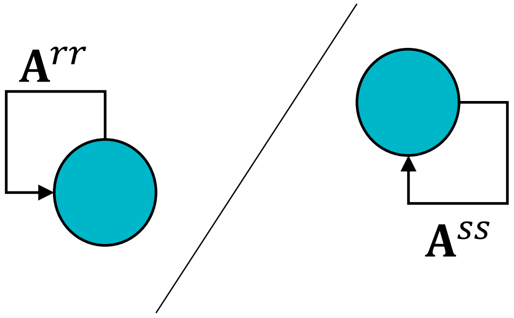
Single region models

- The mathematics is the same for the national and single region IO models.
- But single region models are an exposition of the spatial versatility of input-output models.
- Single region models are often used to highlight the economic qualities of a specific region.
- One way to regionalize tables is to use regional supply percentages.
- Single region models are location-specific; impacts to other regions or economies are not modeled.

Interregional models

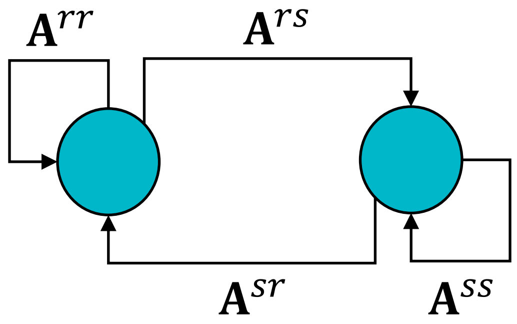
Interregional models

Single-region model



Two or many-region model

“Isard approach”



$$\mathbf{x} = \begin{pmatrix} \mathbf{x}^r \\ \mathbf{x}^s \end{pmatrix} \qquad \mathbf{Z} = \begin{pmatrix} \mathbf{Z}^{rr} & \boxed{\mathbf{Z}^{rs}} \\ \boxed{\mathbf{Z}^{sr}} & \mathbf{Z}^{ss} \end{pmatrix} \qquad \mathbf{f} = \begin{pmatrix} \mathbf{f}^r \\ \mathbf{f}^s \end{pmatrix}$$

Relatively more difficult to capture
than intraregional blocks

Interregional models

REGIONALIZATION APPROACHES

	Survey	**Non-survey**
Pros	More accurate	Less data-intensive
Cons	Resource-intensive	Heavy assumptions
Examples	Input purchases from local vs. other regions Sales to local vs. other regions	Location quotients Supply percentages Fabrication effects Gravity model

Interregional models

Basic structure (2 regions, 2 sectors)

$$\mathbf{Z} = \begin{pmatrix} \mathbf{Z^{rr}} & \mathbf{Z^{rs}} \\ \mathbf{Z^{sr}} & \mathbf{Z^{ss}} \end{pmatrix} = \left(\begin{array}{cc|cc} z_{11}^{rr} & z_{12}^{rr} & z_{11}^{rs} & z_{12}^{rs} \\ z_{21}^{rr} & z_{22}^{rr} & z_{21}^{rs} & z_{22}^{rs} \\ \hline z_{11}^{sr} & z_{12}^{sr} & z_{11}^{ss} & z_{12}^{ss} \\ z_{21}^{sr} & z_{22}^{sr} & z_{21}^{ss} & z_{22}^{ss} \end{array}\right) \qquad \mathbf{x} = \begin{pmatrix} \mathbf{x}^{r} \\ \mathbf{x^{s}} \end{pmatrix} = \left(\begin{array}{c} x_1^r \\ x_2^r \\ \hline x_1^s \\ x_2^s \end{array}\right) \qquad \mathbf{f} = \begin{pmatrix} \mathbf{f}^{r} \\ \mathbf{f^{s}} \end{pmatrix} = \left(\begin{array}{c} f_1^r \\ f_2^r \\ \hline f_1^s \\ f_2^s \end{array}\right)$$

Per usual,

$$\mathbf{A^{rr}} = \mathbf{Z^{rr}}(\hat{\mathbf{x}}^{r})^{-1} \qquad \mathbf{A^{ss}} = \mathbf{Z^{ss}}(\hat{\mathbf{x}}^{s})^{-1}$$

$$\mathbf{A^{sr}} = \mathbf{Z^{sr}}(\hat{\mathbf{x}}^{r})^{-1} \qquad \mathbf{A^{rs}} = \mathbf{Z^{rs}}(\hat{\mathbf{x}}^{s})^{-1}$$

$$\mathbf{Z^{rr}} = \mathbf{A^{rr}}\mathbf{x}^{r} \qquad \mathbf{Z}^{ss} = \mathbf{A}^{ss}\mathbf{x}^{s}$$

$$\mathbf{Z^{sr}} = \mathbf{A^{sr}}\mathbf{x}^{r} \qquad \mathbf{Z^{rs}} = \mathbf{A}^{rs}\mathbf{x}^{s}$$

Interregional models

Basic structure (2 regions, 2 sectors)

$$\mathbf{x^r} = \mathbf{Z^{rr}} + \mathbf{Z^{rs}} + \mathbf{f^r} \longrightarrow x_i^r = \underbrace{z_{i1}^{rr} + \cdots + z_{in}^{rr}}_{\text{Intra-regional, inter-industry sales}} + \underbrace{z_{i1}^{rs} + \cdots + z_{in}^{rr}}_{\text{Inter-regional, inter-industry sales}} + \underbrace{f_i^r}_{\text{Intraregional sales to final demand}}$$

$$\mathbf{x^s} = \mathbf{Z^{sr}} + \mathbf{Z^{ss}} + \mathbf{f^s}$$

Final form:

$$\mathbf{x^r} = (\mathbf{I} - \mathbf{A^{rr}})^{-1}\mathbf{f^r} + (\mathbf{I} - \mathbf{A^{rr}})^{-1}\mathbf{A}^{\mathbf{r}s}\mathbf{x^s}$$

$$\mathbf{x^s} = (\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{f^s} + (\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A^{sr}}\mathbf{x^r}$$

Interregional models

Basic structure (2 regions, 2 sectors)

$$\mathbf{x}^{\mathbf{r}} = (\mathbf{I} - \mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{f}^{\mathbf{r}} + (\mathbf{I} - \mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{r}s}\mathbf{x}^{\mathbf{s}}$$
$$\mathbf{x}^{\mathbf{s}} = (\mathbf{I} - \mathbf{A}^{ss})^{-1}\boxed{\mathbf{f}^{\mathbf{s}}} + (\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{\mathbf{sr}}\mathbf{x}^{\mathbf{r}}$$

Assume for now that $\mathbf{f}^{\mathbf{s}} = \mathbf{0}$

$$\mathbf{x}^{\mathbf{r}} = (\mathbf{I} - \mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{f}^{\mathbf{r}} + (\mathbf{I} - \mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{r}s}\mathbf{x}^{\mathbf{s}}$$

$$\mathbf{x}^{\mathbf{s}} = (\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{\mathbf{sr}}\mathbf{x}^{\mathbf{r}}$$

What happens when final demand for region r changes?

$$\boxed{\mathbf{f}^{\mathbf{r}}}$$

Initial effect

$$\boxed{\mathbf{x}^{\mathbf{r}}} = (\mathbf{I} - \mathbf{A}^{\mathbf{rr}})^{-1}\boxed{\mathbf{f}^{\mathbf{r}}}$$

Intraregional transfer

$$\boxed{\mathbf{x}^{\mathbf{s}}} = (\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{\mathbf{sr}}\boxed{\mathbf{x}^{\mathbf{r}}}$$

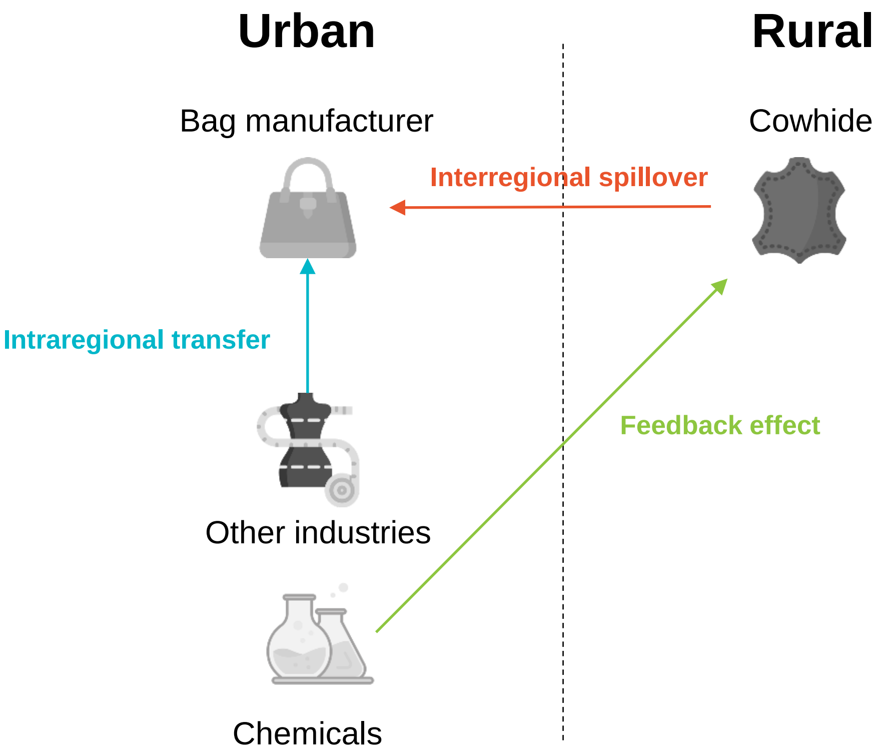
Interregional spillover

$$\boxed{\mathbf{x}^{\mathbf{r}}} = (\mathbf{I} - \mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{r}s}\boxed{\mathbf{x}^{\mathbf{s}}}$$

Feedback effect

Interregional models

Interregional linkages



What happens when final demand for region r changes?

$\boxed{\mathbf{f}^{\mathbf{r}}}$ **Initial effect**

$\boxed{\mathbf{x}^{\mathbf{r}}} = (\mathbf{I} - \mathbf{A}^{\mathbf{rr}})^{-1}\boxed{\mathbf{f}^{\mathbf{r}}}$ **Intraregional transfer**

$\boxed{\mathbf{x}^{\mathbf{s}}} = (\mathbf{I} - \mathbf{A}^{ss})^{-1}\mathbf{A}^{\mathbf{sr}}\boxed{\mathbf{x}^{\mathbf{r}}}$ **Interregional spillover**

$\boxed{\mathbf{x}^{\mathbf{r}}} = (\mathbf{I} - \mathbf{A}^{\mathbf{rr}})^{-1}\mathbf{A}^{\mathbf{rs}}\boxed{\mathbf{x}^{\mathbf{s}}}$ **Feedback effect**

Interregional models

For an economy with many regions from 1 to p,

$$\mathbf{x} = \left[\begin{pmatrix} \mathbf{I} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{I} \end{pmatrix} - \begin{pmatrix} \mathbf{A}^{\mathbf{11}} & \cdots & \mathbf{A}^{\mathbf{1p}} \\ \vdots & \ddots & \vdots \\ \mathbf{A}^{\mathbf{p1}} & \cdots & \mathbf{A}^{\mathbf{pp}} \end{pmatrix} \right]^{-\mathbf{1}} \begin{pmatrix} \mathbf{f}^{\mathbf{1}} \\ \vdots \\ \mathbf{f}^{\mathbf{p}} \end{pmatrix}$$

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f}$$

Some Applications

Japan's interregional spillovers from final demand in all sectors, 2005

Output required from region in *rows* to support demand in *columns*

(Unit: ¥1 billion)

Final demand region / Production inducement region	Hokkaido	Tohoku	Kanto	Chubu	Kinki	Chugoku	Shikoku	Kyushu	Okinawa
Hokkaido		1074 (0.982)	4689 (0.691)	1281 (0.765)	1425 (0.762)	435 (0.602)	176 (0.760)	591 (1.167)	43 (2.217)
Tohoku	1093 (1)		12078 (0.844)	2272 (0.773)	2399 (0.822)	850 (0.760)	419 (0.953)	1302 (1.355)	86 (2.707)
Kanto	6791 (1)	14317 (1)		23940 (1.082)	22173 (1.080)	9400 (1.115)	4590 (1.482)	14086 (1.971)	1031 (2.274)
Chubu	1673 (1)	2941 (1)	22118 (1)		10827 (1.035)	3323 (0.962)	1526 (1.245)	5002 (1.786)	314 (2.511)
Kinki	1870 (1)	2919 (1)	20533 (1)	10465 (1)		5160 (0.967)	2768 (1.318)	6047 (1.493)	374 (2.270)
Chugoku	723 (1)	1118 (1)	8428 (1)	3455 (1)	5338 (1)		1480 (1.276)	4125 (1.553)	171 (4.207)
Shikoku	231 (1)	439 (1)	3097 (1)	1226 (1)	2099 (1)	1160 (1)		1083 (1.246)	52 (2.846)
Kyushu	506 (1)	960 (1)	7146 (1)	2801 (1)	4051 (1)	2656 (1)	869 (1)		349 (2.415)
Okinawa	19 (1)	32 (1)	453 (1)	125 (1)	165 (1)	41 (1)	18 (1)	145 (1)	

Region with the largest interregional demand for all other regions' supply

Region with the largest interregional demand

Region with the largest interregional supply to support all other regions' demand

Okinawa depends more on Chugoku (i.e., former induces more production in the latter) than the reverse

Note 1: Numbers at the top of the table indicate induced domestic products in all industries in regions at the side of the table induced through final demand in the regions at the top of the table

Note 2: If numbers at the bottom of the table are the induction amount 1 when regions with lower region codes are at the top of the table, the opposite coefficient can be found in the opposite case.
For example, the value at the intersection of Kinki at the top of the table and Kanto at the side of the table is divided by the value at the intersection of Kanto at the top of the table and Kinki at the side of the table.

Source: Research and Statistics Department Economic and Industrial Policy Bureau Ministry of Economy, Trade and Industry (Japan). 2010. *2005 Inter-Regional Input-Output Table: A Debrief Report*. March. https://www.meti.go.jp/english/statistics/tyo/tiikiio/pdf/2005report.pdf

Some Applications

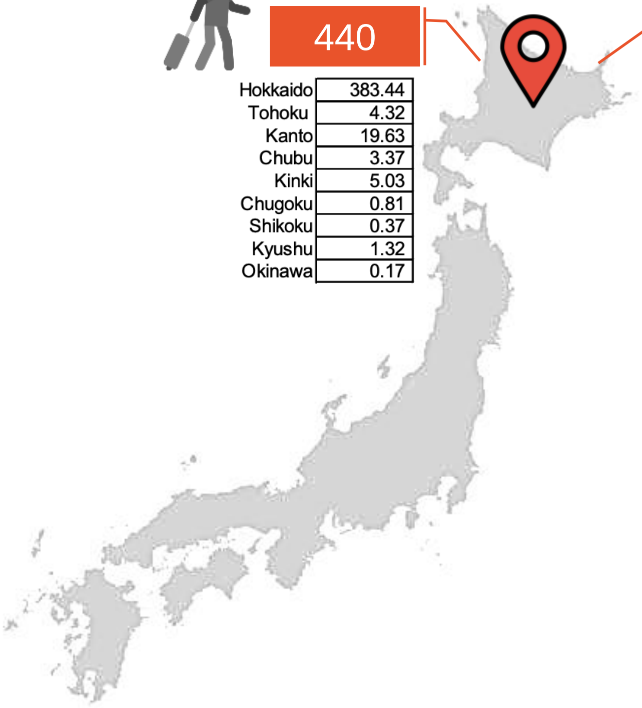
Impacts of Consumption by Foreign Visitors who Travel to Hokkaido, 2005

(100 million Japanese Yen ¥)

Direct demand from foreign visitor traveling to Hokkaido

440

Hokkaido	383.44
Tohoku	4.32
Kanto	19.63
Chubu	3.37
Kinki	5.03
Chugoku	0.81
Shikoku	0.37
Kyushu	1.32
Okinawa	0.17



Domestic production

709.57

	Induced domestic products
Hokkaido	**537.86**
Tohoku	**17.25**
Kanto	**96.88**
Chubu	**18.04**
Kinki	**23.08**
Chugoku	**6.54**
Shikoku	**2.76**
Kyushu	**6.65**
Okinawa	**0.51**
Total	**709.57**

Goods

117.33

Services

592.24

By goods and construction and service industries

	Goods	Construction and service industries
Hokkaido	46.87	490.99
Tohoku	7.75	9.50
Kanto	33.29	63.59
Chubu	10.15	7.89
Kinki	10.08	13.00
Chugoku	4.20	2.34
Shikoku	1.68	1.08
Kyushu	3.18	3.47
Okinawa	0.13	0.38
Total	117.33	592.24

Source: Research and Statistics Department Economic and Industrial Policy Bureau Ministry of Economy, Trade and Industry (Japan). 2010. *2005 Inter-Regional Input-Output Table: A Debrief Report*. March. https://www.meti.go.jp/english/statistics/tyo/tiikiio/pdf/2005report.pdf

SECTION SUMMARY

Interregional models

- The fundamental structure is still the same for the national, single-region, and two or many-region (interregional) IO models. That is, $\mathbf{x} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{f}$.

- Interregional linkages however are made explicit in interregional IO models, thereby providing estimates for spillovers and feedback effects.

- However, estimating interregional flows are relatively more challenging than intraregional flows. Nonsurvey estimates are less than ideal but are cost-effective.

Multiregional models *(Chenery-Moses)*

Chenery-Moses Model (Multiregional Model)

- Essentially, also an *interregional* model.
- Only in this case, **interregional trade coefficients** are used to estimate interregional flows ("Chenery-Moses" approach).

Chenery, Hollis B. 1953. "Regional Analysis," in Hollis B. Chenery, Paul G. Clark and Vera Cao Pinna (eds.), The Structure and Growth of the Italian Economy. Rome: US Mutual Security Agency, pp. 97–129.

Moses, Leon N. 1955. "The Stability of Interregional Trading Patterns and Input-Output Analysis," American Economic Review, 45, 803–832.

Chenery-Moses Model (Multiregional Model)

Table of Interregional Trade
(or Shipments) of Commodity i

Shipping Region	Receiving Region: 1	2	$\cdots$	s	$\cdots$	p
1						
2						
$\vdots$						
r				z_i^{rs}		
$\vdots$						
p						
Total						

Ratio of commodity i supplies from region r to all the supplies of i in region s

$$\boxed{c_i^{rs} = \frac{z_i^{rs}}{T_i^s}}$$

$$T_i^s = z_i^{1s} + z_i^{2s} + \cdots + z_i^{rs} + \cdots + z_i^{ps}$$

Source: Miller, R., & Blair, P. (2009). Input-Output Analysis: Foundations and Extensions (2nd ed.). Cambridge: Cambridge University Press.

Chenery-Moses Model (Multiregional Model)

Assuming interregional shipments for commodities i to n are available,

$$c_1^{rs} = \frac{z_1^{rs}}{T_1^s}$$

$$\vdots$$

$$= \begin{pmatrix} c_1^{rs} \\ \vdots \\ c_i^{rs} \\ \vdots \\ c_n^{rs} \end{pmatrix} = \mathbf{c}^{rs}$$

$$\vdots$$

$$c_n^{rs} = \frac{z_n^{rs}}{T_n^s}$$

Diagonalizing,

$$\hat{\mathbf{c}}^{rs} = \begin{bmatrix} c_1^{rs} & 0 & \cdots & 0 \\ 0 & c_2^{rs} & & \\ \vdots & & & \\ 0 & 0 & \cdots & c_n^{rs} \end{bmatrix}$$

Supply ratios from r to s for all commodities / sectors (1 to n)

Chenery-Moses Model (Multiregional Model)

Produce trade proportions matrices for all pairs of regions 1 to p:

Receiving region

Supplying region

	1	2	…	s	…	p
1	$\hat{\mathbf{c}}^{11}$	$\hat{\mathbf{c}}^{12}$	…	$\hat{\mathbf{c}}^{1s}$	…	$\hat{\mathbf{c}}^{1p}$
2	$\hat{\mathbf{c}}^{21}$	$\hat{\mathbf{c}}^{22}$	…	$\hat{\mathbf{c}}^{2s}$	…	$\hat{\mathbf{c}}^{2p}$
⋮	⋮	⋮	…	⋮	…	⋮
r	$\hat{\mathbf{c}}^{r1}$	$\hat{\mathbf{c}}^{r1}$	…	$\hat{\mathbf{c}}^{rs}$	…	$\hat{\mathbf{c}}^{rp}$
⋮	⋮	⋮	…	⋮	…	⋮
p	$\hat{\mathbf{c}}^{p1}$	$\hat{\mathbf{c}}^{p2}$	…	$\hat{\mathbf{c}}^{ps}$	…	$\hat{\mathbf{c}}^{pp}$

$= \mathbf{C}$

$$\hat{\mathbf{c}}^{rs} = \begin{bmatrix} c_1^{rs} & 0 & \cdots & 0 \\ 0 & c_2^{rs} & & \\ \vdots & & & \\ 0 & 0 & \cdots & c_n^{rs} \end{bmatrix}$$

Chenery-Moses Model (Multiregional Model)

Consider a two-sector, two-region case:

$$\begin{matrix} \mathbf{A}^{rr} & \mathbf{A}^{rs} \\ \mathbf{A}^{sr} & \mathbf{A}^{ss} \end{matrix}$$

Input coefficients of sectors in r $\quad \mathbf{A}^{r} \quad \mathbf{A}^{s} \quad$ Input coefficients of sectors in s

Example:

Coal fired power plant in region r

80%

Technical coefficient of electricity distributor in r for its purchases of raw energy

Wind power plant in region s

20%

Electricity distributor in region r

$$a^{\cdot r}_{raw,distribution} = 0.7$$

$$a^{rr}_{raw,distribution} = 0.80\ x\ 0.7 = 0.56$$

$$a^{sr}_{raw,distribution} = 0.20\ x\ 0.7 = 0.14$$

$$a^{\cdot r}_{raw,distribution} = 0.56 + 0.14 = 0.7$$

Chenery-Moses Model (Multiregional Model)

Consider a two-sector, two-region case:

Input coefficients of sectors in r

$\mathbf{A}^r$

$\mathbf{A}^s$

Input coefficients of sectors in s

Chenery-Moses approach:

$$\mathbf{A}^{\mathbf{rr}} = \hat{\mathbf{c}}^{\mathbf{rr}}\mathbf{A}^{\mathbf{r}} \qquad \mathbf{A}^{\mathbf{rs}} = \hat{\mathbf{c}}^{\mathbf{rs}}\mathbf{A}^{\mathbf{s}}$$

$$\mathbf{A}^{\mathbf{sr}} = \hat{\mathbf{c}}^{\mathbf{sr}}\mathbf{A}^{\mathbf{r}} \qquad \mathbf{A}^{\mathbf{ss}} = \hat{\mathbf{c}}^{\mathbf{ss}}\mathbf{A}^{\mathbf{s}}$$

$$\hat{\mathbf{c}}^{rs}\mathbf{A}^{s} = \begin{pmatrix} c_1^{rs}a_{11}^{s} & c_1^{rs}a_{12}^{s} \\ c_2^{rs}a_{21}^{s} & c_2^{rs}a_{22}^{s} \end{pmatrix}$$

Say c_1^{rs} = 0.60, then sector *1* and *2* in region *s* both source 60% of its product *1* inputs from region *r*.

$$\hat{\mathbf{c}}^{sr}\mathbf{A}^{r} = \begin{pmatrix} c_1^{sr}a_{11}^{r} & c_1^{sr}a_{12}^{r} \\ c_2^{sr}a_{21}^{r} & c_2^{sr}a_{22}^{r} \end{pmatrix}$$

Say c_2^{sr} = 0.50, then sector *1* and *2* in region *r* both source 50% of its product *2* inputs from region *s*.

Chenery-Moses Model (Multiregional Model)

Consider a two-sector, two-region case:

$$\begin{array}{cc} \mathbf{f}^{rr} & \mathbf{f}^{rs} \\ \mathbf{f}^{sr} & \mathbf{f}^{ss} \end{array}$$

Final demand in r $\quad \mathbf{f}^{r} \qquad \mathbf{f}^{s}$ $\quad$ Final demand in s

Chenery-Moses approach:

$$\mathbf{f}^{\mathbf{rr}} = \hat{\mathbf{c}}^{\mathbf{rr}}\mathbf{f}^{\mathbf{r}} \qquad \mathbf{f}^{\mathbf{rs}} = \hat{\mathbf{c}}^{\mathbf{rs}}\mathbf{f}^{\mathbf{s}}$$

$$\mathbf{f}^{\mathbf{sr}} = \hat{\mathbf{c}}^{\mathbf{sr}}\mathbf{f}^{\mathbf{r}} \qquad \mathbf{f}^{\mathbf{ss}} = \hat{\mathbf{c}}^{\mathbf{ss}}\mathbf{f}^{\mathbf{s}}$$

► $\hat{\mathbf{c}}^{rs}\mathbf{f}^{s} = \begin{pmatrix} c_1^{rs} f_1^{s} \\ c_2^{rs} f_2^{s} \end{pmatrix}$

Assumes that trade proportions are the same for final consumers and businesses. That is, sector *1* in region *s* sources 60% of its inputs of product 1 from region *r*; final consumers in region *s* also obtain 60% of their product 1 purchases from region *r*.

Chenery-Moses Model (Multiregional Model)

Consider a two-sector, two-region case:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^r & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^s \end{bmatrix}, \ \mathbf{C} = \begin{bmatrix} \hat{\mathbf{c}}^{rr} & \hat{\mathbf{c}}^{rs} \\ \hat{\mathbf{c}}^{sr} & \hat{\mathbf{c}}^{ss} \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} \mathbf{x}^r \\ \mathbf{x}^s \end{bmatrix}, \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{f}^r \\ \mathbf{f}^s \end{bmatrix}$$

$\mathbf{A}$ is different in that $\mathbf{A}^r$ and $\mathbf{A}^s$ represent input coefficients of each sector j from both regions.
More precisely, $\mathbf{A}^{\mathbf{r}} = \mathbf{A}^{(r+s),r}$ and $\mathbf{A}^{\mathbf{s}} = \mathbf{A}^{(r+s),s}$

$\mathbf{f}$ is also different in that f contains final consumption of each product i from both regions.
More precisely, $\mathbf{f}^r = \mathbf{f}^{(r+s),r}$ and $\mathbf{f}^s = \mathbf{f}^{(r+s),s}$

Chenery-Moses Model (Multiregional Model)

Consider a two-sector, two-region case:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}^r & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^s \end{bmatrix}, \mathbf{C} = \begin{bmatrix} \hat{\mathbf{c}}^{rr} & \hat{\mathbf{c}}^{rs} \\ \hat{\mathbf{c}}^{sr} & \hat{\mathbf{c}}^{ss} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \mathbf{x}^r \\ \mathbf{x}^s \end{bmatrix}, \text{ and } \mathbf{f} = \begin{bmatrix} \mathbf{f}^r \\ \mathbf{f}^s \end{bmatrix}$$

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f}$$

$$\mathbf{x} = \mathbf{CAx} + \mathbf{Cf}$$

$$\mathbf{x} - \mathbf{CAx} = \mathbf{CAx} + \mathbf{Cf} - \mathbf{CAx}$$

$$\mathbf{x} - \mathbf{CAx} = \mathbf{Cf}$$

$$(\mathbf{I} - \mathbf{CA})\mathbf{x} = \mathbf{Cf}$$

$$(\mathbf{I} - \mathbf{CA})^{-1}(\mathbf{I} - \mathbf{CA})\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf}$$

$$\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf}$$

Miller, Ronald E., and Peter D. Blair. Input-Output Analysis: Foundations and Extensions. 2nd ed. Cambridge: Cambridge University Press, 2009.

Chenery-Moses Model (Multiregional Model)

Numerical illustration using People's Republic of China's (PRC) Multiregional Input-Output Table for 2000

Table constructed by IDE-JETRO (originally 30 sectors, 8 regions); See sources below.

Regional Aggregation

3-Region Aggregation	Regions	Provinces and Municipalities
North	Northeast	Heilongjiang, Jilin, Liaoning
	North	Beijing, Tianjin, Hebei, Shandong
South	South	Hainan, Guangdong, Fujian
	Central	Hunan, Jiangxi, Hubei, Henan, Anhui, Shanxi
	East	Jiangsu, Shanghai, Zhejiang
Rest of PRC	Northwest	Xinjiang, Qinghai, Gansu, Ningxia, Shaanxi, Inner Mongolia
	Southwest	Tibet, Sichuan, Yunnan, Guizhou, Guangxi, Chongqing

Sectoral Aggregation

3-Sector Aggregation	Industry Sectors
Natural Resources	agriculture
	mining & processing
Manufacturing & Construction	light industry
	energy industry
	heavy industry & chemical industry
	construction
Services & Other Sectors	transportation & telecommunications services
	commercial services
	other

Sources:
Okamoto, Nabuhiro and Takeo Ihara (eds.). 2005. Spatial Structure and Regional Development in China. An Interregional Input-Output Approach. Basingstoke, UK: Palgrave Macmillan.

Miller, R., & Blair, P. (2009). Input-Output Analysis: Foundations and Extensions (2nd ed.). Cambridge: Cambridge University Press.

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Chenery-Moses Model (Multiregional Model)

Numerical illustration using People's Republic of China's (PRC) Multiregional Input-Output Table for 2000

Given the following **interregional trade** data (CNY 10,000):

Source	Destination		
	North	South	Rest of PRC
North			
Natural Resources	8,442	1,480	67
Manuf. & Const.	23,826	4,092	330
Services	6,403	634	33
South			
Natural Resources	847	13,198	296
Manuf. & Const.	4,868	43,874	1,610
Services	445	10,681	256
ROC			
Natural Resources	63	305	5,028
Manuf. & Const.	345	1,355	9,662
Services	36	240	3,570

One can easily derive c-ratios for each sector (n=3)

Source	Destination		
	North	South	Rest of PRC
North			
Natural Reso	0.903	0.099	0.012
Manuf. & Cor	0.820	0.083	0.028
Services	0.930	0.055	0.009
South			
Natural Reso	0.091	0.881	0.055
Manuf. & Cor	0.168	0.890	0.139
Services	0.065	0.924	0.066
ROC			
Natural Reso	0.007	0.020	0.933
Manuf. & Cor	0.012	0.027	0.833
Services	0.005	0.021	0.925

Example:

$$c_i^{rs} = \frac{z_i^{rs}}{T_i^s}$$

$$c_{Services}^{North,South} = \frac{z_{Services}^{North,South}}{T_{Services}^{South}} = \frac{634}{634 + 10{,}681 + 240} = 0.055$$

Chenery-Moses Model (Multiregional Model)

Numerical illustration using People's Republic of China's (PRC) Multiregional Input-Output Table for 2000

Vectors **c** for all region pairs:

Diagonalizing each vector **c** to form the matrix **C**,

	North			South			Rest of PRC		
	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services
North									
Natural Resources	0.903	-	-	0.099	-	-	0.012	-	-
Manuf. & Const.	-	0.820	-	-	0.083	-	-	0.028	-
Services	-	-	0.930	-	-	0.055	-	-	0.009
South									
Natural Resources	0.091	-	-	0.881	-	-	0.055	-	-
Manuf. & Const.	-	0.168	-	-	0.890	-	-	0.139	-
Services	-	-	0.065	-	-	0.924	-	-	0.066
ROC									
Natural Resources	0.007	-	-	0.020	-	-	0.933	-	-
Manuf. & Const.	-	0.012	-	-	0.027	-	-	0.833	-
Services	-	-	0.005	-	-	0.021	-	-	0.925

Chenery-Moses Model (Multiregional Model)

Numerical illustration using People's Republic of China's (PRC) Multiregional Input-Output Table for 2000

Provided an **A** matrix for each region:

	North			South			Rest of PRC		
	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services
North									
Natural Resources	0.113	0.142	0.030						
Manuf. & Const.	0.173	A^{N} 0.455	0.240						
Services	0.046	0.085	0.128						
South									
Natural Resources				0.136	0.127	0.038			
Manuf. & Const.				0.150	A^{S} 0.484	0.246			
Services				0.043	0.089	0.133			
ROC									
Natural Resources							0.146	A^{R} 0.160	0.035
Manuf. & Const.							0.126	0.383	0.231
Services							0.040	0.110	0.120

Chenery-Moses Model (Multiregional Model)

Numerical illustration using People's Republic of China's (PRC) Multiregional Input-Output Table for 2000

One can easily multiply **CA** matrices to get:

CA matrix	North			South			Rest of PRC		
	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services
North - Nat. Res.	0.1020	0.1278	0.0271	0.0134	0.0125	0.0038	0.0018	0.0020	0.0004
North - Manuf. & Const.	0.1417	$A^{N,N}$ [illegible]	0.1967	0.0124	$A^{N,S}$ [illegible]	0.0204	0.0036	$A^{N,R}$ [illegible]	0.0066
North - Services	0.0426	0.0789	0.1188	0.0023	0.0049	0.0073	0.0003	0.0009	0.0010
South - Nat. Res.	0.0102	0.0128	0.0027	0.1196	0.1115	0.0339	0.0080	0.0088	0.0019
South - Manuf. & Const.	0.0290	$A^{S,N}$ [illegible]	0.0402	0.1335	$A^{S,S}$ [illegible]	0.2186	0.0174	$A^{S,R}$ [illegible]	0.0321
South - Services	0.0030	0.0055	0.0083	0.0394	0.0821	0.1232	0.0026	0.0073	0.0079
RoPRC - Nat. Res.	0.0008	0.0010	0.0002	0.0028	0.0026	0.0008	0.1358	0.1494	0.0327
RoPRC - Manuf. & Const.	0.0021	$A^{R,N}$ [illegible]	0.0028	0.0041	$A^{R,S}$ [illegible]	0.0068	0.1047	$A^{R,R}$ [illegible]	0.1924
RoPRC - Services	0.0002	0.0004	0.0007	0.0009	0.0018	0.0028	0.0366	0.1022	0.1108

Recall the **A** matrix as:

	North			South			Rest of PRC		
	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services
North									
Natural Resources	0.113	0.142	0.030						
Manuf. & Const.	0.173	0.455	0.240						
Services	0.046	0.085	0.128						
South									
Natural Resources				0.136	0.127	0.038			
Manuf. & Const.				0.150	0.484	0.246			
Services				0.043	0.089	0.133			
ROC									
Natural Resources							0.146	0.160	0.035
Manuf. & Const.							0.126	0.383	0.231
Services							0.040	0.110	0.120

Chenery-Moses Model (Multiregional Model)

Numerical illustration using People's Republic of China's (PRC) Multiregional Input-Output Table for 2000

Finally, inverting the (**I**-**CA**) matrix (per usual Leontief equation):

(I-CA)-1 matrix	North			South			Rest of PRC		
	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services
North - Nat. Res.	1.160933	0.256687	0.095936	0.031191	0.054810	0.026819	0.007237	0.015447	0.008465
North - Manuf. & Const.	0.298253	1.729569	0.403640	0.058142	0.157262	0.086849	0.019579	0.052095	0.031948
North - Services	0.083727	0.169478	1.176839	0.012350	0.029700	0.022090	0.003457	0.008955	0.006007
South - Nat. Res.	0.034415	0.067759	0.032660	1.181372	0.254521	0.111540	0.023399	0.046273	0.024149
South - Manuf. & Const.	0.121690	0.292340	0.163089	0.321483	1.921367	0.502117	0.074143	0.199238	0.122667
South - Services	0.019622	0.043956	0.030815	0.083977	0.193250	1.193661	0.014134	0.036888	0.026680
RoPRC - Nat. Res.	0.003706	0.007703	0.003906	0.008081	0.015674	0.008000	1.195926	0.279571	0.105117
RoPRC - Manuf. & Const.	0.010181	0.024355	0.013633	0.017368	0.048039	0.026749	0.206355	1.570179	0.349448
RoPRC - Services	0.002194	0.004985	0.003305	0.004479	0.011119	0.008337	0.073132	0.192516	1.169445

This Leontief inverse matrix can now be used to evaluate total impacts of exogenous demands f in any region.

Chenery-Moses Model (Multiregional Model)

Numerical illustration using People's Republic of China's (PRC) Multiregional Input-Output Table for 2000

As an example, separately evaluate the case when each demand for each sector in each region is "100" (=100x10,000 CNY).

Regional impacts of 1,000,000 CNY worth of demand for each product in every region, PRC

(in 10,000 CNY)

Impacts by region ▼	North			South			Rest of PRC		
	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services	Natural Resources	Manuf. & Const.	Services
North	140.22	181.02	156.83	24.26	39.60	21.84	5.30	15.86	6.63
South	30.31	73.20	32.85	141.74	214.88	168.67	19.35	57.54	28.23
Rest of PRC	2.72	6.72	3.07	5.80	12.57	7.47	137.79	171.22	150.54
Total	173.25	260.94	192.74	171.80	267.05	197.98	162.44	244.62	185.40

SECTION SUMMARY

Multiregional models

- The "Chenery-Moses" approach enables interregional input-output analysis when users only have two types of information:
    a. Trade between and among regions; and
    b. Input coefficients in each region

- This approach however also carries the same assumptions as regional supply percentages (*that all users of the same product obtain supplies from other regions in the same proportions*).

- Nevertheless, the analysis can be informative as it follows trade patterns of regional economies. *For example, changes in the density of trade flows between any two pairs of regions (or economies) can change interregional input coefficients, and therefore the magnitude of potential spillovers.*

Hands-on exercises

Please open the Exercises tab of the excel workbook for Day 2 Input-Output Models at the Regional Level

The Dataset

- Economies (G=6)
  - People's Republic of China (PRC)
  - Kazakhstan (KAZ)
  - Kyrgyz Republic (KGZ)
  - Pakistan (PAK)
  - Mongolia (MON)
  - Rest of the world (RoW)
- Sectors (N=5)
  - Primary
  - Low-technology manufacturing sectors
  - Medium-to-high technology sectors
  - Business services
  - Personal and public services

The Dataset

- There are 6 given input coefficient **(A)** matrices for year 2019.

Economy name (CODE)

	People's Republic of China (PRC)				
	Primary	Low-technology	Medium- to high·	Business servic	Personal and pu
Primary	0.113	0.097	0.069	0.012	0.005
Low-technology manufa	0.118	0.291	0.115	0.071	0.093
Medium- to high-technol	0.101	0.214	0.464	0.079	0.192
Business services	0.079	0.144	0.142	0.269	0.205
Personal and public serv	0.014	0.020	0.022	0.040	0.133

Purchasing sectors

Selling sectors

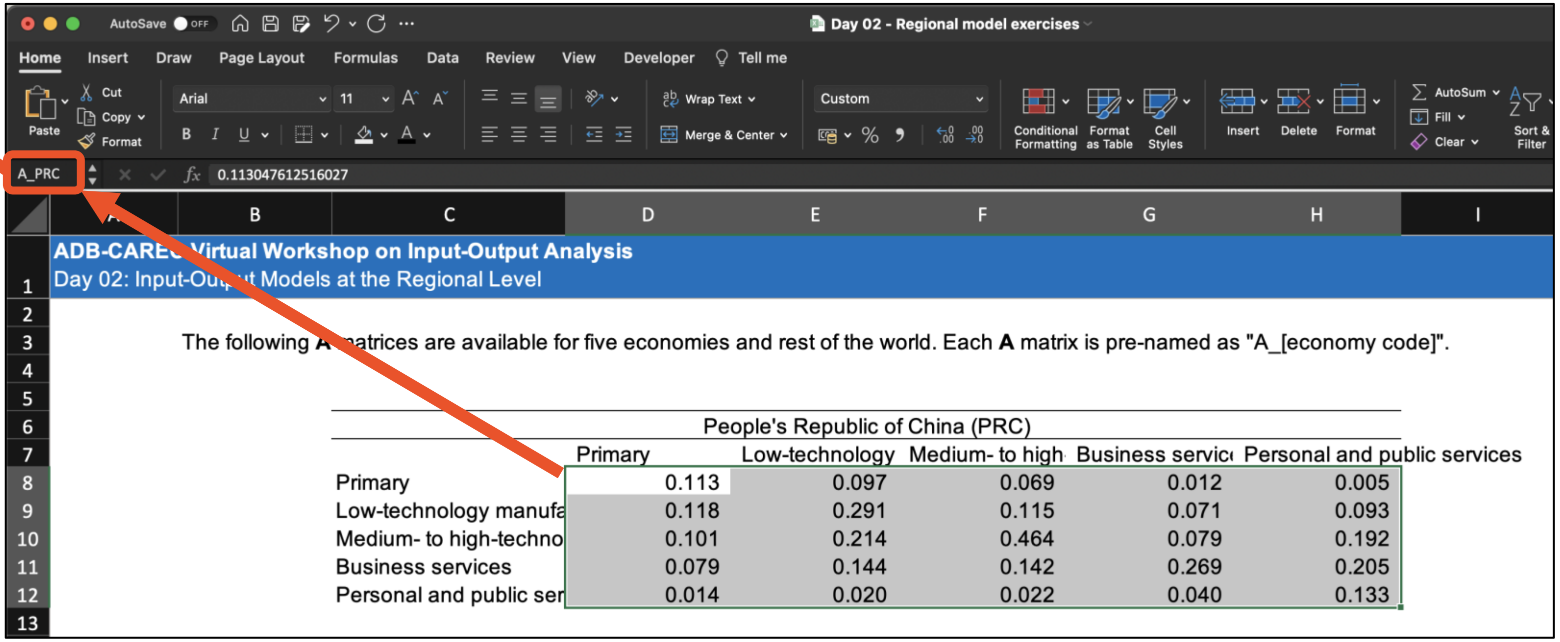
A matrix (undistributed)

Respective arrays are pre-named.

The Dataset

- A matrices are named for convenience.
  - A_PRC
  - A_KAZ
  - A_KGZ
  - A_MON
  - A_PAK
  - A_ROW

Screenshot:



	People's Republic of China (PRC)				
	Primary	Low-technology	Medium- to high	Business servic	Personal and public services
Primary	0.113	0.097	0.069	0.012	0.005
Low-technology manufa	0.118	0.291	0.115	0.071	0.093
Medium- to high-techno	0.101	0.214	0.464	0.079	0.192
Business services	0.079	0.144	0.142	0.269	0.205
Personal and public ser	0.014	0.020	0.022	0.040	0.133

The Dataset

- Gross trade (in million US$) within and across economies are also given for 2019.

(in million $)		Receiving economy						
Supplying economy & sector		Kazakhstan	Kyrgyz Republic	Mongolia	Pakistan	People's Republic of China	Rest of the world	Total
Kazakhstan	Primary	25,459	96	1	0	3,290	36,347	65,194
	Low-technology manufa	31,125	301	25	0	261	1,223	32,936
	Medium- to high-technol	13,077	369	1	0	5,102	11,479	30,029
	Business services	120,693	17	9	0	1,622	5,949	128,290
	Personal and public ser	35,100	0	0	0	26	76	35,202
Kyrgyz Republic	Primary	124	3,215	0	0	32	425	3,797
	Low-technology manufa	52	3,552	0	0	15	310	3,930
	Medium- to high-technol	68	1,159	0	0	6	938	2,172
	Business services	27	6,571	1	0	84	845	7,528
	Personal and public ser	0	1,704	0	1	19	177	1,901
Mongolia	Primary	0	-	2,307	-	4,109	2,315	8,731
	Low-technology manufa	2	0	5,683	0	183	256	6,124
	Medium- to high-technol	0	0	773	-	107	242	1,122
	Business services	2	0	6,755	0	608	547	7,912
	Personal and public ser	0	0	2,501	0	11	30	2,542
Pakistan	Primary	9	0	0	84,628	157	804	85,597
	Low-technology manufa	5	1	4	95,396	1,071	16,377	112,855
	Medium- to high-technol	9	2	1	33,962	633	1,722	36,330
	Business services	2	0	1	154,800	283	2,332	157,419
	Personal and public ser	1	1	0	41,656	145	2,048	43,850
People's Republic of China	Primary	149	33	131	167	2,439,701	25,485	2,465,667
	Low-technology manufa	1,868	346	805	2,924	11,003,925	750,060	11,759,928
	Medium- to high-technol	5,328	872	1,470	7,518	11,210,572	1,643,389	12,869,149
	Business services	705	164	288	696	9,736,102	206,384	9,944,339
	Personal and public ser	1	11	7	230	6,479,073	15,073	6,494,395
Rest of the world	Primary	3,565	115	221	8,258	430,979	7,808,665	8,251,803
	Low-technology manufa	7,707	670	1,409	7,764	269,735	24,564,320	24,851,605
	Medium- to high-technol	23,346	1,872	2,586	16,092	1,291,393	21,343,850	22,679,139
	Business services	8,590	653	2,047	3,841	407,177	56,826,604	57,248,912
	Personal and public ser	70	163	245	3,933	82,106	22,598,516	22,685,034

Gross output (x) by sector and economy

Pre-named as x_all

Item 1: Calculate T_i^s for all i and s.

T is the total amount of product i received by economy s.

D102

	A	B	C	D	E	F	G	H	I	J
1	**ADB-CAREC Virtual Workshop on Input-Output Analysis** Day 02: Input-Output Models at the Regional Level									
96		**Exercises**								
97		Please fill out the yellow-shaded areas.								
98										
99		1. Calculate the total amount *T* of product *i* to *n* (i.e., from primary to personal and public services) received in each economy/region.								
100										
101				Kazakhstan	Kyrgyz Republic	Mongolia	Pakistan	People's Republ	Rest of the world	
102			Primary							
103			Low-technology manufacturing							
104		Total	Medium- to high-technology manufacturing							
105			Business services							
106			Personal and public services							
107										
108										
109		2. Calculate the elements in the proportions matrix **C**. These elements should refer to economy / region *s*'s (column) share to the total consumption of regio								
110		As a check, proportions cannot exceed 100%. Each column should total to 100% multiplied by the number of sectors (n = 5).								
111										
112				Kazakhstan	Kyrgyz Republic	Mongolia	Pakistan	People's Republ	Rest of the world	
113			Primary							
114			Low-technology manufacturing							
115		Kazakhstan	Medium- to high-technology manufacturing							
116			Business services							

Item 2: Calculate $c_i^{rs} = \frac{z_i^{rs}}{T_i^s}$ for all i and pairs of s.

c is the proportion of product i received by economy s from r.

D113

ADB-CAREC Virtual Workshop on Input-Output Analysis
Day 02: Input-Output Models at the Regional Level

2. Calculate the elements in the proportions matrix **C**. These elements should refer to economy / region *s*'s (column) share to the total consumption of regio
As a check, individual proportions cannot exceed 100%. Each column should total to 500% [or 100% multiplied by the number of sectors (n = 5)].

		Kazakhstan	Kyrgyz Republic	Mongolia	Pakistan	People's Republ	Rest of the world
Kazakhstan	Primary						
	Low-technology manufacturing						
	Medium- to high-technology manufacturing						
	Business services						
	Personal and public services						
Kyrgyz Republic	Primary						
	Low-technology manufacturing						
	Medium- to high-technology manufacturing						
	Business services						
	Personal and public services						
Mongolia	Primary						
	Low-technology manufacturing						
	Medium- to high-technology manufacturing						
	Business services						
	Personal and public services						
Pakistan	Primary						
	Low-technology manufacturing						
	Medium- to high-technology manufacturing						

Item 2: Calculate $c_i^{rs} = \frac{z_i^{rs}}{T_i^s}$ for all i and pairs of s.

c is the proportion of product i received by economy s from r.

			Kazakhstan	Kyrgyz Republi	Mongolia	Pakistan	People's Reput	Rest of the world
		Primary	0.87	0.03	0.00	0.00	0.00	0.00
		Low-technology manu	0.76	0.06	0.00	0.00	0.00	0.00
$\mathbf{c}^{KAZ,KAZ}$	Kazakhstan	Medium- to high-techn	0.31	0.09	0.00	0.00	0.00	0.00
		Business services	0.93	0.00	0.00	0.00	0.00	0.00
		Personal and public se	1.00	0.00	0.00	0.00	0.00	0.00
		Primary	0.00	0.93	0.00	0.00	0.00	0.00
		Low-technology manu	0.00	0.73	0.00	0.00	0.00	0.00
$\mathbf{c}^{KGZ,KAZ}$	Kyrgyz Republi	Medium- to high-techn	0.00	0.27	0.00	0.00	0.00	0.00
		Business services	0.00	0.89	0.00	0.00	0.00	0.00
		Personal and public se	0.00	0.91	0.00	0.00	0.00	0.00
		Primary	0.00	0.00	0.87	0.00	0.00	0.00
		Low-technology manu	0.00	0.00	0.72	0.00	0.00	0.00
$\mathbf{c}^{MON,KAZ}$	Mongolia	Medium- to high-techn	0.00	0.00	0.16	0.00	0.00	0.00
		Business services	0.00	0.00	0.74	0.00	0.00	0.00
		Personal and public se	0.00	0.00	0.91	0.00	0.00	0.00
		Primary	0.00	0.00	0.00	0.91	0.00	0.00
		Low-technology manu	0.00	0.00	0.00	0.90	0.00	0.00
$\mathbf{c}^{PAK,KAZ}$	Pakistan	Medium- to high-techn	0.00	0.00	0.00	0.59	0.00	0.00
		Business services	0.00	0.00	0.00	0.97	0.00	0.00
		Personal and public se	0.00	0.00	0.00	0.91	0.00	0.00
		Primary	0.01	0.01	0.05	0.00	0.85	0.00
	People's	Low-technology manu	0.05	0.07	0.10	0.03	0.98	0.03
$\mathbf{c}^{PRC,KAZ}$	Republic of	Medium- to high-techn	0.13	0.20	0.30	0.13	0.90	0.07
	China	Business services	0.01	0.02	0.03	0.00	0.96	0.00
		Personal and public se	0.00	0.01	0.00	0.01	0.99	0.00
		Primary	0.12	0.03	0.08	0.09	0.15	0.99
	Rest of the	Low-technology manu	0.19	0.14	0.18	0.07	0.02	0.97
$\mathbf{c}^{ROW,KAZ}$	world	Medium- to high-techn	0.56	0.44	0.54	0.28	0.10	0.93
		Business services	0.07	0.09	0.22	0.02	0.04	1.00
		Personal and public se	0.00	0.09	0.09	0.09	0.01	1.00

$c_{primary}^{KAZ,KGZ}$ = % of Kazakhstan in the supply of primary products to Kyrgyz Republic (about 2.8%)

$c_{medhigh}^{PRC,ROW}$ = % of PRC in the supply of primary products to rest of the world (about 7.1%)

Item 3-4: Assumptions of the Chenery-Moses model

- Recall that the Chenery-Moses model operationalizes the interregional input-output model if the following data are given:
  - ☑ Input coefficients matrix at the regional (or national level)
  - ☑ Cross-regional (or international) trade flows are provided by sector/product

Assumes that:

- Input coefficients at the national level reflect the average purchases of all subregions from all sources.
- All purchasing sectors in the economy (or region) *s* purchase the same proportion of their supply of commodity *i* from region *r*.

Item 5: Arranging the A matrices

A_KAZ
A_KGZ
A_MON
A_PAK
A_PRC
A_ROW

Item 5: Arranging the A matrices

Simply use name of the array to retrieve the corresponding **A** matrix. Do not forget to use CTRL+SHIFT+ENTER.

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5. Arrange all **A** matrices in diagonal blocks (i.e., place zeroes elsewhere). Rename this matrix as "A_global".

	B	C	D	E	F	G	H	I	J	K	L	M
157					Kazakhstan					Kyrgyz Republic		
158			Primary	Low-technology	Medium- to high-	Business service	Personal and pu	Primary	Low-technology	Medium- to hig	Business ser	Personal and Pr
159	Kazakhstan	Primary	0.13	0.06	0.16	0.01	0.01	-	-	-	-	-
160		Low-technology manufa	0.03	0.07	0.01	0.05	0.09	-	-	-	-	-
161		Medium- to high-techno	0.09	0.15	0.11	0.05	0.06	-	-	-	-	-
162		Business services	0.21	0.16	0.20	0.24	0.27	-	-	-	-	-
163		Personal and public ser	0.00	0.00	0.00	0.00	0.01	-	-	-	-	-
164	Kyrgyz Republi	Primary	-	-	-	-	-					
165		Low-technology manufa	-	-	-	-	-					
166		Medium- to high-techno	-	-	-	-	-					
167		Business services	-	-	-	-	-					
168		Personal and public ser	-	-	-	-	-					
169	Mongolia	Primary	-	-	-	-	-	-	-	-	-	-
170		Low-technology manufa	-	-	-	-	-	-	-	-	-	-
171		Medium- to high-techno	-	-	-	-	-	-	-	-	-	-
172		Business services	-	-	-	-	-	-	-	-	-	-
173		Personal and public ser	-	-	-	-	-	-	-	-	-	-
174	Pakistan	Primary	-	-	-	-	-	-	-	-	-	-
175		Low-technology manufa	-	-	-	-	-	-	-	-	-	-
176		Medium- to high-techno	-	-	-	-	-	-	-	-	-	-
177		Business services	-	-	-	-	-	-	-	-	-	-
178		Personal and public ser	-	-	-	-	-	-	-	-	-	-
179	People's Republic of China	Primary	-	-	-	-	-	-	-	-	-	-
180		Low-technology manufa	-	-	-	-	-	-	-	-	-	-
181		Medium- to high-techno	-	-	-	-	-	-	-	-	-	-
182		Business services										

Item 6: Preparing the **C** matrix

- From item 2, you have:

$$\begin{array}{llllll}
\mathbf{c}^{KAZ,KAZ} & = \begin{bmatrix}0.869\\0.764\\0.313\\0.928\\0.998\end{bmatrix} & \mathbf{c}^{KAZ,MON} & \mathbf{c}^{KAZ,PAK} & \mathbf{c}^{KAZ,PRC} & \mathbf{c}^{KAZ,ROW} \\
\mathbf{c}^{KGZ,KAZ} & & \mathbf{c}^{KGZ,MON} & \mathbf{c}^{KGZ,PAK} & \mathbf{c}^{KGZ,PRC} & \mathbf{c}^{KGZ,ROW} \\
\mathbf{c}^{MON,KAZ} & \mathbf{c}^{MON,KGZ} & \mathbf{c}^{MON,MON} & \mathbf{c}^{MON,PAK} & \mathbf{c}^{MON,PRC} & \mathbf{c}^{MON,ROW} \\
\mathbf{c}^{PAK,KAZ} & \mathbf{c}^{PAK,KGZ} & \mathbf{c}^{PAK,MON} & \mathbf{c}^{PAK,PAK} & \mathbf{c}^{PAK,PRC} & \mathbf{c}^{PAK,ROW} \\
\mathbf{c}^{PRC,KAZ} & \mathbf{c}^{PRC,KGZ} & \mathbf{c}^{PRC,MON} & \mathbf{c}^{PRC,PAK} & \mathbf{c}^{PRC,PRC} & \mathbf{c}^{PRC,ROW} \\
\mathbf{c}^{ROW,KAZ} & \mathbf{c}^{ROW,KGZ} & \mathbf{c}^{ROW,MON} & \mathbf{c}^{ROW,PAK} & \mathbf{c}^{ROW,PRC} & \mathbf{c}^{ROW,ROW}
\end{array}$$

Item 6: Preparing the **C** matrix

Diagonalizing each **c**-vector,

$$\hat{\mathbf{c}}^{KAZ,KAZ} = \begin{bmatrix} 0.869 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0.998 \end{bmatrix}$$

$$\mathbf{C} = \begin{pmatrix} \hat{\mathbf{c}}^{KAZ,KAZ} & & & \hat{\mathbf{c}}^{KAZ,PAK} & \hat{\mathbf{c}}^{KAZ,PRC} & \hat{\mathbf{c}}^{KAZ,ROW} \\ \hat{\mathbf{c}}^{KGZ,KAZ} & & & \hat{\mathbf{c}}^{KGZ,PAK} & \hat{\mathbf{c}}^{KGZ,PRC} & \hat{\mathbf{c}}^{KGZ,ROW} \\ \hat{\mathbf{c}}^{MON,KAZ} & \hat{\mathbf{c}}^{MON,KGZ} & \hat{\mathbf{c}}^{MON,MON} & \hat{\mathbf{c}}^{MON,PAK} & \hat{\mathbf{c}}^{MON,PRC} & \hat{\mathbf{c}}^{MON,ROW} \\ \hat{\mathbf{c}}^{PAK,KAZ} & \hat{\mathbf{c}}^{PAK,KGZ} & \hat{\mathbf{c}}^{PAK,MON} & \hat{\mathbf{c}}^{PAK,PAK} & \hat{\mathbf{c}}^{PAK,PRC} & \hat{\mathbf{c}}^{PAK,ROW} \\ \hat{\mathbf{c}}^{PRC,KAZ} & \hat{\mathbf{c}}^{PRC,KGZ} & \hat{\mathbf{c}}^{PRC,MON} & \hat{\mathbf{c}}^{PRC,PAK} & \hat{\mathbf{c}}^{PRC,PRC} & \hat{\mathbf{c}}^{PRC,ROW} \\ \hat{\mathbf{c}}^{ROW,KAZ} & \hat{\mathbf{c}}^{ROW,KGZ} & \hat{\mathbf{c}}^{ROW,MON} & \hat{\mathbf{c}}^{ROW,PAK} & \hat{\mathbf{c}}^{ROW,PRC} & \hat{\mathbf{c}}^{ROW,ROW} \end{pmatrix}$$

Item 6: Preparing the **C** matrix

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6. Arrange **c** vectors into a **C** matrix. For ease, other submatrices are filled out for you. Try to fill in only blank matrices (in yellow shade). Rename the entire matrix as "C_matrix".

	B	C	D	E	F	G	H	I	J	K	L	M	N	O
194			Kazakhstan					Kyrgyz Republic						
195		Code for economies:	1	1	1	1	1	2	2	2	2	2	3	3
196		Code for sectors:	1	2	3	4	5	1	2	3	4	5	1	2
197	Kazakhstan	1						0.03	-	-	-	-	0.00	-
198		2						-	0.06	-	-	-	-	0.0
199		3						-	-	0.09	-	-	-	-
200		4						-	-	-	0.00	-	-	-
201		5						-	-	-	-	0.00	-	-
202	Kyrgyz Republic	1	0.00	-	-	-	-	0.93	-	-	-	-	0.00	-
203		2	-	0.00	-	-	-	-	0.73	-	-	-	-	0.0
204		3	-	-	0.00	-	-	-	-	0.27	-	-	-	-
205		4	-	-	-	0.00	-	-	-	-	0.89	-	-	-
206		5	-	-	-	-	0.00	-	-	-	-	0.91	-	-
207	Mongolia	1	0.00	-	-	-	-	-	-	-	-	-	0.87	-
208		2	-	0.00	-	-	-	-	0.00	-	-	-	-	0.7
209		3	-	-	0.00	-	-	-	-	0.00	-	-	-	-
210		4	-	-	-	0.00	-	-	-	-	0.00	-	-	-
211		5	-	-	-	-	0.00	-	-	-	-	0.00	-	-
212	Pakistan	1	0.00	-	-	-	-	0.00	-	-	-	-	0.00	-
213		2	-	0.00	-	-	-	-	0.00	-	-	-	-	0.0
214		3	-	-	0.00	-	-	-	-	0.00	-	-	-	-
215		4	-	-	-	0.00	-	-	-	-	0.00	-	-	-
216		5	-	-	-	-	0.00	-	-	-	-	0.00	-	-
217	People's Republic of China	1	0.01	-	-	-	-	0.01	-	-	-	-	0.05	-
218		2	-	0.05	-	-	-	-	0.07	-	-	-	-	0.1
219		3	-	-	0.13	-	-	-	-	0.20	-	-	-	-
220		4	-	-	-	0.01	-	-	-	-	0.02	-	-	-
221		5	-	-	-	-	0.00	-	-	-	-	0.01	-	-
222	Rest of the	1	0.12	-	-	-	-	0.03	-	-	-	-	0.08	-
223		2	-	0.19	-	-	-	-	0.14	-	-	-	-	0.1

Item 7: Leontief inverse

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		D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
Rest of the world	2	-	0.19	-	-	-	-	0.14	-	-	-	-	0.18	-	-	-	-	0.07	-
	3	-	-	0.56	-	-	-	-	0.44	-	-	-	-	0.54	-	-	-	-	0.28
	4	-	-	-	0.07	-	-	-	-	0.09	-	-	-	-	0.22	-	-	-	-
	5	-	-	-	-	0.00	-	-	-	-	0.09	-	-	-	-	0.09	-	-	-

7. Calculate the Leontief inverse matrix, $(I\text{-}CA)^{-1}$ of the multiregional input-output model. Feel free to use the named arrays for each component. Rename the resulting matrix as "L_MRIO."

		Kazakhstan					Kyrgyz Republic					Mongolia					Pakistan		
		Primary	Low-technology r	Medium- to high-t	Business service	Personal and put	Primary	Low-technology r	Medium- to higl	Business ser	Personal and	Primary	Low-technolo	Medium- to hi	Business ser	Personal and	Primary	Low-technolo	Medium- to hi Business
Kazakhstan	Primary																		
	Low-technology manufacturing																		
	Medium- to high-technology manufacturing																		
	Business services																		
	Personal and public services																		
Kyrgyz Republi	Primary																		
	Low-technology manufacturing																		
	Medium- to high-technology manufacturing																		
	Business services																		
	Personal and public services																		
Mongolia	Primary																		
	Low-technology manufacturing																		
	Medium- to high-technology manufacturing																		
	Business services																		
	Personal and public services																		
Pakistan	Primary																		
	Low-technology manufacturing																		
	Medium- to high-technology manufacturing																		
	Business services																		
	Personal and public services																		
People's Republic of China	Primary																		
	Low-technology manufacturing																		
	Medium- to high-technology manufacturing																		
	Business services																		
	Personal and public services																		
Rest of the world	Primary																		
	Low-technology manufacturing																		
	Medium- to high-technology manufacturing																		
	Business services																		
	Personal and public services																		

8. Use the following formula to derive the *undistributed* final demand in each economy (or region).

$$\mathbf{f} = \mathbf{C}^{-1}(\mathbf{I} - \mathbf{CA})\mathbf{x},$$

Economy/Regic	Sector	Undistributed final demand in the economy/region (in million $) (i.e., final consumption in economy of outputs produced from all regions)	Memo: Distributed final demand fc Final demand f	Summation vector
	Primary	12,528.8	20,368	1

Item 7: Leontief inverse

```
L_MRIO = MINVERSE(MUNIT(30)-MMULT(C_matrix,A_global))
```

$$(\mathbf{I} - \mathbf{C}\mathbf{A})^{-1}$$

Item 8: Deriving the final demand

$$\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf}$$

$$\mathbf{f} = \mathbf{C}^{-1}(\mathbf{I} - \mathbf{CA})\mathbf{x},$$

$$\underbrace{\texttt{MMULT(MINVERSE(C\_matrix),MMULT(MUNIT(30)-MMULT(C\_matrix,A\_global),x\_all))}}_{\textbf{f\_vector}}$$

Item 8: Deriving the final demand

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8. Use the following formula to derive the *undistributed* final demand in each economy (or region).

$$\mathbf{f} = \mathbf{C}^{-1}(\mathbf{I} - \mathbf{CA})\mathbf{x},$$

Economy/Regio	Sector	Undistributed final demand in the economy/region (in million $) (i.e., final consumption in economy of outputs produced from all regions)
Kazakhstan	Primary	
	Low-technology manufacturing	
	Medium- to high-technology manufacturing	
	Business services	
	Personal and public services	
Kyrgyz Republic	Primary	
	Low-technology manufacturing	
	Medium- to high-technology manufacturing	
	Business services	
	Personal and public services	
Mongolia	Primary	
	Low-technology manufacturing	
	Medium- to high-technology manufacturing	
	Business services	
	Personal and public services	
Pakistan	Primary	
	Low-technology manufacturing	
	Medium- to high-technology manufacturing	
	Business services	
	Personal and public services	
People's Republic of China	Primary	
	Low-technology manufacturing	
	Medium- to high-technology manufacturing	
	Business services	
	Personal and public services	
Rest of the world	Primary	
	Low-technology manufacturing	
	Medium- to high-technology manufacturing	
	Business services	

Item 9: Implementing the baseline MRIO model

$$\mathbf{x} = \boxed{(\mathbf{I} - \mathbf{CA})^{-1}}\boxed{\mathbf{Cf}}$$

```
=MMULT(L_MRIO,MMULT(C_matrix,f_vector))
```

Item 9: Implementing the baseline MRIO model

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9. To check if the model set up is correct, evaluate the estimated final demand in the Chenery-Moses (multiregional) model. Check its consistency with the baseline output levels.

$$\mathbf{x} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{Cf}.$$

(in million $)

Economy/Regio	Sector	Model output (x)	Baseline output (	Difference
	Primary		65,194	65,194
	Low-technology manufacturing		32,936	32,936
Kazakhstan	Medium- to high-technology manufacturing		30,029	30,029
	Business services		128,290	128,290
	Personal and public services		35,202	35,202
	Primary		3,797	3,797
	Low-technology manufacturing		3,930	3,930
Kyrgyz Republic	Medium- to high-technology manufacturing		2,172	2,172
	Business services		7,528	7,528
	Personal and public services		1,901	1,901
	Primary		8,731	8,731
	Low-technology manufacturing		6,124	6,124
Mongolia	Medium- to high-technology manufacturing		1,122	1,122
	Business services		7,912	7,912
	Personal and public services		2,542	2,542
	Primary		85,597	85,597
	Low-technology manufacturing		112,855	112,855
Pakistan	Medium- to high-technology manufacturing		36,330	36,330
	Business services		157,419	157,419
	Personal and public services		43,850	43,850
	Primary		2,465,667	2,465,667
People's	Low-technology manufacturing		11,759,928	11,759,928
Republic of	Medium- to high-technology manufacturing		12,869,149	12,869,149
China	Business services		9,944,339	9,944,339
	Personal and public services		6,494,395	6,494,395
	Primary		8,251,803	8,251,803
	Low-technology manufacturing		24,851,605	24,851,605
Rest of the world	Medium- to high-technology manufacturing		22,679,139	22,679,139

Format as Table

Item 10: Experimenting with the model

SUM | fx | =

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world	Business services	57,248,912	57,248,912	-
	Personal and public ser	22,685,034	22,685,034	-

10. Use the model above to evaluate ***only*** the final demand in Kazakhstan for primary sector products (i.e., place zeroes elsewhere).
That is, retain the corresponding value of Kazakhstan's primary sector in the (undistributed) final demand vector and assume all else to be zero.

(in million $)

Economy/Regic	Sector	Final demand	Output impacts
Kazakhstan	Primary		
	Low-technology manufacturing		
	Medium- to high-technology manufacturing		
	Business services		
	Personal and public services		
Kyrgyz Republi	Primary		
	Low-technology manufacturing		
	Medium- to high-technology manufacturing		
	Business services		
	Personal and public services		
Mongolia	Primary		
	Low-technology manufacturing		
	Medium- to high-technology manufacturing		
	Business services		
	Personal and public services		
Pakistan	Primary		
	Low-technology manufacturing		
	Medium- to high-technology manufacturing		
	Business services		
	Personal and public services		
People's Republic of China	Primary		
	Low-technology manufacturing		
	Medium- to high-technology manufacturing		
	Business services		
	Personal and public services		
Rest of the world	Primary		
	Low-technology manufacturing		
	Medium- to high-technology manufacturing		
	Business services		
	Personal and public services		

Conditional Formatting

How much is the local (or domestic) impact to Kazakhstan? -

Item 10: Experimenting with the model

Outputs required to satisfy current final demands in Kazakhstan for primary products.

(in million $)

Economy/Regic	Sector	Final demand	Output impacts
Kazakhstan	Primary	12,529	12,420
	Low-technology manufa	-	441
	Medium- to high-techno	-	447
	Business services	-	3,288
	Personal and public ser	-	38
Kyrgyz Republi	Primary	-	99
	Low-technology manufa	-	4
	Medium- to high-techno	-	6
	Business services	-	29
	Personal and public ser	-	0
Mongolia	Primary	-	0
	Low-technology manufa	-	0
	Medium- to high-techno	-	0
	Business services	-	0
	Personal and public ser	-	0
Pakistan	Primary	-	6
	Low-technology manufa	-	1
	Medium- to high-techno	-	1
	Business services	-	1
	Personal and public ser	-	0
People's Republic of China	Primary	-	136
	Low-technology manufa	-	185
	Medium- to high-techno	-	533
	Business services	-	185
	Personal and public ser	-	28
Rest of the world	Primary	-	2,253
	Low-technology manufa	-	586
	Medium- to high-techno	-	1,634
	Business services	-	1,295
	Personal and public ser	-	69

Impact to domestic economy = $16.6 billion

Impact to external economies = $7.0 billion

Item 11: Disaggregating the impacts per final demand

$$\hat{\mathbf{x}} = \boxed{(\mathbf{I} - \mathbf{CA})^{-1}}\boxed{\mathbf{C}}\boxed{\hat{\mathbf{f}}}$$

```
=MMULT(L_MRIO,
       MMULT(C_matrix,
       IF(ROW(1:30)=TRANSPOSE(ROW(1:30)),f_vector,0)))
```

Notice that the first column of the resulting matrix is the same as the column vector result in the previous item.

Item 11: Disaggregating the impacts per final demand

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11. Evaluate all final demands simultaneously. Hint: Diagonalizing the (undistributed) final demand vector (f_vector), before premultiplying with the Leontief inverse (array: L_MRIO) and C_matrix.

$\hat{\mathbf{x}} = (\mathbf{I} - \mathbf{CA})^{-1}\mathbf{C}\hat{\mathbf{f}}$.

(in million $)

		Kazakhstan					Kyrgyz Republic					
Supplying econ	Sector	Primary	Low-technology n	Medium- to high-t	Business service	Personal and pub	Primary	Low-technology m	Medium- to high	Business serv	Personal and	Pr
Kazakhstan	Primary											
	Low-technology manufacturing											
	Medium- to high-technology manufacturing											
	Business services											
	Personal and public services											
Kyrgyz Republic	Primary											
	Low-technology manufacturing											
	Medium- to high-technology manufacturing											
	Business services											
	Personal and public services											
Mongolia	Primary											
	Low-technology manufacturing											
	Medium- to high-technology manufacturing											
	Business services											
	Personal and public services											
Pakistan	Primary											
	Low-technology manufacturing											
	Medium- to high-technology manufacturing											
	Business services											
	Personal and public services											
People's Republic of China	Primary											
	Low-technology manufacturing											
	Medium- to high-technology manufacturing											

Item 11: Disaggregating the impacts per final demand

(in million $)

Source of Demand (columns)

Supplying ecc	Sector	Kazakhstan					Kyrgyz Republic					Mongolia				
		Primary	Low-technology	Medium- to higl	Business servi	Personal and p	Primary	Low-technology	Medium- to hi	Business se	Personal an	Primary	Low-technol	Medium- to	Business s	Personal an
Kazakhstan	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
Kyrgyz Repub	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
Mongolia	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
Pakistan	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
People's Republic of China	Primary											101	191	139	34	23
	Low-technology manu											77	1,067	328	123	82
	Medium- to high-techn											147	895	1,550	246	165
	Business services											72	450	384	257	79
	Personal and public s											10	70	65	21	17
Rest of the world	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															

Supplying ecc	Sector	Pakistan					People's Republic of China					Rest of the world				
		Primary	Low-technol	Medium- to	Business s	Personal and	Primary	Low-technolc	Medium- to hig	Business servi	Personal and p	Primary	Low-technolog	Medium- to high	Business servic	Personal and publ
Kazakhstan	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
Kyrgyz Repub	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
Mongolia	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
Pakistan	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
People's Republic of China	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															
Rest of the world	Primary															
	Low-technology manu															
	Medium- to high-techn															
	Business services															
	Personal and public s															

How much production was contributed by People's Republic of China (PRC) to satisfy the final demand in Mongolia?

PRODUCER > ROW > WHICH ECONOMY? > PRC ; WHICH SECTOR? > ALL

DEMAND > COLUMN > WHICH ECONOMY? > MON ; WHICH SECTOR? > ALL

Answer: $6.593 billion

Item 12: Scenario Analysis

Find the change (△f) in final demands. Name this vector as "`f_change`".

B434 | 11B. For simplicity, the results above (in no. 11) are aggregated to the economy-level.

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12. Consider the following scenario. Final demands in each economy and region were projected to have changed in the following year due to the pandemic. The rates of change are provided below.
For simplicity, assume that the growth/decline in each economy's final demands is uniform across all types of products. What would be the overall impact of this scenario?

Economy/region	% Growth (Decline) in Final Demand
Kazakhstan	-3%
Kyrgyz Republic	-9%
Mongolia	-5%
Pakistan	-1%
People's Republic of Cl	2%
Rest of the world	-3%

Source: World Bank Development Indicators (accessed February 25, 2022)

Determine the change in final demand levels from the baseline. Use the undistributed final demand.

Economy	Sector	% Change in fir	Change in final demand, 2019 - 2020 (in million $)
Kazakhstan	Primary	-3%	
	Low-technology manufa	-3%	
	Medium- to high-techno	-3%	
	Business services	-3%	
	Personal and public sei	-3%	
Kyrgyz Republi	Primary	-9%	
	Low-technology manufa	-9%	
	Medium- to high-techno	-9%	
	Business services	-9%	
	Personal and public sei	-9%	
Mongolia	Primary	-5%	
	Low-technology manufa	-5%	
	Medium- to high-techno	-5%	
	Business services	-5%	
	Personal and public sei	-5%	
Pakistan	Primary	-1%	
	Low-technology manufa	-1%	
	Medium- to high-techno	-1%	
	Business services	-1%	

Item 12: Scenario Analysis

Find the changes in output (△x) using the Chenery-Moses model.

D535

	B	C	D	E	F	G	H	I
1	REC Virtual Workshop on Input-Output Analysis nput-Output Models at the Regional Level							
530	Diagonalize the computed change in final demands, then evaluate the total impacts to all regions using the Chenery-Moses (multiregional) model.							
531	Hint: Use the same formula as no. 11, but replace f_vector with f_change.							
532								
533					Kazakhstan			
534	Supplying econ	Sector	Primary	Low-technology n	Medium- to high-t	Business service	Personal and pub	Primary
535		Primary						
536		Low-technology manufacturing						
537	Kazakhstan	Medium- to high-technology manufacturing						
538		Business services						
539		Personal and public services						
540		Primary						
541		Low-technology manufacturing						
542	Kyrgyz Republic	Medium- to high-technology manufacturing						
543		Business services						
544		Personal and public services						
545		Primary						
546		Low-technology manufacturing						
547	Mongolia	Medium- to high-technology manufacturing						
548		Business services						

Item 12: Scenario Analysis

Key results:

- Sum of the resulting matrix = change in output in the world economy (or $3.5 trillion) from 2019-2020.
- PRC's modest growth in demand cushioned some negative impacts in other economies, but not enough to pull production up to a positive territory.
- Kyrgyz experienced the highest loss in output in % terms.

Output impacts from decline in final demands in each economy/region

	Baseline output ($ mln.)	Change in output ($ mln.)	% Change
Kazakhstan	291,651	(7,167)	-2.5%
Kyrgyz Republic	19,327	(1,358)	-7.0%
Mongolia	26,432	(710)	-2.7%
Pakistan	436,051	(4,770)	-1.1%
People's Republic of China	43,533,477	586,114	1.3%
Rest of the world	135,716,493	(4,095,674)	-3.0%

Summary

- Input-output models may be adjusted to reflect production characteristics of a specified economic territory.
- Data are more limited at lower levels (e.g., subnational territories, cities, etc.). Nonsurvey techniques partially address this difficulty but must be applied with caution.
- Economic spillovers may be estimated through interregional models.