General Structure of Multiplier Analysis

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Overview

- Examine key summary measures known as **multipliers** derived from input-output models to estimate the effects of exogenous changes on the following:

1. Output
2. Income and Employment
3. Value-Added

Recap: On Input-Output Model

- The Input-Output Model can assess the effect of changes in elements that are exogenous to the model of an economy.
- Miller and Blair (2009) presented several numerical illustrations when assumed changes in final-demand elements are interpreted through the Leontief inverse.

Impact Analysis	Vs.	Forecasting
• conducted or explored when exogenous changes occur due to the actions of an "impacting agent". • are expected to occur in the short run.		• conducted or explored for broader changes. • are expected to occur in the long run.

Multiplier analysis using input-output tables

$$multiplier = \frac{total\ effects}{initial\ effects}$$

- In an IO model that is open with respect to households, total effects can be either **direct** and **indirect** effects. When incorporated, these are simple multipliers.
- In an IO model that is closed with respect to households, total effects can be **direct**, **indirect**, and **induced** effects. When incorporated, these are total multipliers.

Output Multipliers

Output Multipliers

- Total additional output needed for production in order to satisfy a dollar's worth of increase in demand for a sectors final products.

$$\boldsymbol{L} = \begin{bmatrix} l_{11} & l_{12} & l_{13} & \cdots & l_{1n} \\ l_{21} & l_{22} & l_{23} & \cdots & l_{2n} \\ l_{31} & l_{32} & l_{33} & \cdots & l_{3n} \\ & & \ddots & & \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nn} \end{bmatrix}$$

Total output induced by \$1 of final demand for sector 1's products

Note that,
Initial effect to output = change in final demand = \$1

Therefore,

$$output\ multiplier\ of\ sector\ 1 = \frac{total\ effects}{initial\ effect}$$

$$= \frac{\sum_{i=1}^{n} l_{i1}}{1}$$

$$= \sum_{i=1}^{n} l_{i1}$$

Simple Output Multipliers

- Incorporated in an IO model that is **open** with respect to households.
- Equal to the ratio of direct and indirect effects to the initial effect.
- Denoted as:

$$m(o)_j = \frac{\sum_{i=1}^{n} l_{ij}}{\$1} = \sum_{i=1}^{n} l_{ij}$$

$$\boldsymbol{m}\,(o) = [m(o)_1 \quad \cdots \quad m(o)_n] = \underbrace{\mathbf{i}' \overbrace{\boldsymbol{L}}^{Sector-demand-to-sector-output-multipliers}}_{sector-demand-to-economy-wide-output\ multipliers}$$

where, $\mathbf{i}' = [1 \quad 1 \quad \cdots \quad 1]$

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Total Output Multipliers

- Incorporated in IO model that is **closed** with respect to households.
- Equal to the ratio of direct, indirect, and **induced** effects to the initial effect
- Denoted as:

$$\bar{\boldsymbol{L}} = [\bar{l}_{ij}] = (\boldsymbol{I} - \bar{\boldsymbol{A}})^{-1} = \begin{bmatrix} \bar{\boldsymbol{L}}_{11} & \bar{\boldsymbol{L}}_{12} \\ \bar{\boldsymbol{L}}_{21} & \bar{\boldsymbol{L}}_{22} \end{bmatrix}$$

- The logic behind $\bar{\boldsymbol{L}}$ is similar to $\boldsymbol{L}$, but with larger effects due to the inclusion of effects to households.

Total Output Multipliers

- The total output multiplier for sector j is

$$\bar{m}(o)_j = \sum_{i=1}^{n+1} \bar{l}_{ij}$$

- In terms of matrix notation,

$$\bar{\boldsymbol{m}}(o) = [\bar{m}(o)_1 \quad \cdots \quad \bar{m}(o)_{n+1}] = \boldsymbol{i}'\bar{\boldsymbol{L}}$$

Example

Using Economy A Input-Output table for 2019

Simple Output Multipliers

Top 5 Sectors in Economy A for 2019

1.70
Coke, refined petroleum, and nuclear fuel

1.69
Public administration and defense; compulsory social security

1.64
Water transport

1.60
Electrical and optical equipment

1.60
Chemicals and chemical products

Income and Employment Multipliers

Income Multipliers

- Income multipliers attempt to identify the impacts of change in new final demand on income received by households.

- Income input coefficients denoted as

$$\boldsymbol{h}'_c = \boldsymbol{h}'\widehat{\boldsymbol{x}}^{-1} = \begin{bmatrix} \frac{h_1}{x_1} & \cdots & \frac{h_n}{x_n} \end{bmatrix} = \begin{bmatrix} a_{n+1,1} & \cdots & a_{n+1,n} \end{bmatrix}$$

- With these, we can compute for the **simple household income multiplier for sector *j***

$$m(h)_j = \frac{\sum_{i=1}^{n} a_{n+1,i} l_{ij}}{\Delta f} = \sum_{i=1}^{n} a_{n+1,i} l_{ij}$$

Income Multipliers

- **Total income effects** or household income multipliers are obtained using a parallel formula

$$\bar{m}(h)_j = \frac{\sum_{i=1}^{n+1} a_{n+1,i}\bar{l}_{ij}}{\Delta f} = \sum_{i=1}^{n+1} a_{n+1,i}\bar{l}_{ij}$$

- To simplify, $\bar{m}(h)_j = \bar{l}_{n+1,j}$
- In matrix notation,

$$\bar{\boldsymbol{m}}(h) = [\bar{m}(h)_1 \quad \cdots \quad \bar{m}(h)_n] = [\boldsymbol{h}_c' \quad a_{n+1,n+1}]\begin{bmatrix}\bar{\boldsymbol{L}}_{11} \\ \bar{\boldsymbol{L}}_{21}\end{bmatrix} = \bar{\boldsymbol{h}}_c'\begin{bmatrix}\bar{\boldsymbol{L}}_{11} \\ \bar{\boldsymbol{L}}_{21}\end{bmatrix} = \bar{\boldsymbol{L}}_{21}$$

Income Multipliers

- **Type I income multiplier** for any sector j, as

$$m(h)_j^I = \frac{\sum_{i=1}^{n} a_{n+1,i} l_{ij}}{a_{n+1,j}} = \frac{m(h)_j}{a_{n+1,j}}$$

- In matrix notation,

$$\boldsymbol{m}(h)^I = \boldsymbol{m}(h)\left(\widehat{\boldsymbol{h}}_c'\right)^{-1} = \boldsymbol{h}_c' \boldsymbol{L}\left(\widehat{\boldsymbol{h}}_c'\right)^{-1}$$

- **Type II income multiplier** for any sector j, as

$$m(h)_j^{II} = \frac{\sum_{i=1}^{n+1} a_{n+1,i} \bar{l}_{ij}}{a_{n+1,j}} = \frac{\bar{m}(h)_j}{a_{n+1,j}} \quad \text{or} \quad m(h)_j^{II} = \frac{\bar{l}_{n+1,j}}{a_{n+1,j}}$$

- In matrix notation,

$$\boldsymbol{m}(h)^{II} = \bar{\boldsymbol{L}}_{21}\left(\widehat{\boldsymbol{h}}_c'\right)^{-1}$$

Example

Using small example of Input-Output model

Employment Multipliers

- Similar types of multipliers can be produced if one is interested in counts of jobs, in physical terms.

Example

Using Economy A Input-Output table for 2019

Simple Employment Multipliers

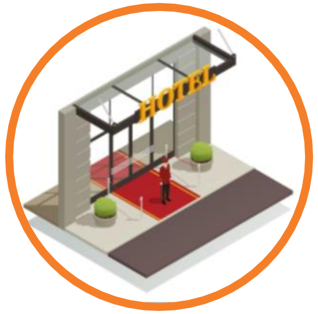
Top 5 Sectors in Economy A for 2019

61.28
Health and social work

61.03
Education

46.87
Public administration and defense; compulsory social security

17.47
Post and telecommunications



16.54
Hotels and restaurants

Type I Employment Multipliers

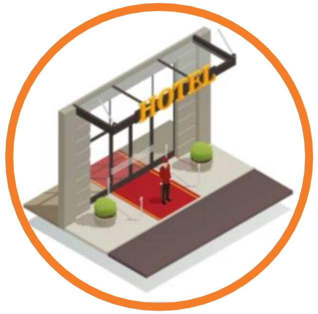
Top 5 Sectors in Economy A for 2019

49.45
Health and social work

34.38
Education

28.77
Public administration and defense; compulsory social security

12.41
Hotels and restaurants



12.26
Post and telecommunications

Value-Added Multipliers

Value-Added Multipliers

- In an input-output table, the gross value-added vector can be used to measure the **total value-added** embodied in each unit of final demand.

- A set of sectoral **value-added coefficients** are required to compute value added multipliers → $\boldsymbol{v}'_c = \boldsymbol{v}'\hat{\boldsymbol{x}}^{-1}$

- The approach is to convert the components of the Leontief inverse matrix from output into value-added terms through value-added coefficients.

- This "multiplier" can then be directly used to evaluate impacts that are more bounded to the value of gross domestic product.

Example

Using Economy A Input-Output table for 2019

Simple Value-Added Multipliers

Top 5 Sectors in Economy A for 2019

0.91
Real estate

0.90
Rubber and plastics

0.89
Basic metals and fabricated metal

0.88
Post and telecommunications

0.88
Other community, social, and personal services

Type I Value-Added Multipliers

Top 5 Sectors in Economy A for 2019

2.13
Public administration and defense; compulsory social security

2.01
Water transport

1.90
Coke, refined petroleum, and nuclear fuel

1.81
Chemicals and chemical products

1.80
Air transport

Thank you!

Reference

Miller, R. E., & Blair, P. D. (2009). Input-output analysis: foundations and extensions. Cambridge university press.

Image Sources

vecteezy.com