Leontief Output Model

CAREC Workshop

April 4-8, 2022

Outline

- Recap of Single Region Input-Output Tables
- Open and Closed Economy Input-Output Tables
- Closing the Input-Output Model with Respect to Households

Quick Quiz

- MCQs based on the presentation
- Please type your answers in the chat box

Recap of Single-Region IO Table

Structure and Interpretation of the IO Table

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
	1st *quadrant*			2nd *quadrant*		
Industry 1	z_{11}	…	z_{1I}	y_{11}	… y_{1Q}	x_1
	Z			**Y**		
Industry i	…	z_{ij}	…	…	y_{iq} …	x_i
	Intermediate Consumption Matrix			**Final Demand Matrix**		
Industry I	z_{I1}	…	z_{II}	y_{I1}	… y_{IQ}	x_I
	3rd *quadrant*			4th *quadrant*		
Imports	…	…	…	…	… …	M
	V					
Value added	…	v_{pj}	…	…	y_{pq} …	Y
	Value Added / Primary Inputs					
Total	x_1	x_j	x_I	C I	G E	

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
	1st quadrant			2nd quadrant		
Industry 1	z_{11}	$\cdots$	z_{1I}	y_{11}	$\cdots$ y_{1Q}	x_1
Industry i	$\cdots$	z_{ij}	$\cdots$	$\cdots$	y_{iq} $\cdots$	x_i
Industry I	z_{I1}	$\cdots$	z_{II}	y_{I1}	$\cdots$ y_{IQ}	x_I
	3rd quadrant			4th quadrant		
Imports	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$ $\cdots$	M
Value added	$\cdots$	v_{pj}	$\cdots$	$\cdots$	y_{pq} $\cdots$	Y
Total	x_1	x_j	x_I	C I	G E	

Z

Intermediate Consumption Matrix

+

V

Value Added / Primary Inputs

Total Production by purchasing industry j = x_j

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
		1st *quadrant*		2nd *quadrant*		
Industry 1	z_{11}	…	z_{1I}	y_{11} …	y_{1Q}	x_1
Industry i	…	z_{ij}	…	… y_{iq}	…	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1} …	y_{IQ}	x_I
		3rd *quadrant*		4th *quadrant*		
Imports	…	…	…	… …	…	M
Value added	…	v_{pj}	…	… y_{pq}	…	Y
Total	x_1	x_j	x_I	C I G	E	

Y

Final Demand Matrix

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
	1st *quadrant*			2nd *quadrant*		
Industry 1	z_{11}	…	z_{1I}	y_{11}	… y_{1Q}	x_1
Industry i	…	z_{ij}	…	…	y_{iq} …	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1}	… y_{IQ}	x_I
	3rd *quadrant*			4th *quadrant*		
Imports	…	…	…	…	… …	
Value added	…	v_{pj}	…	…	y_{pq} …	
Total	x_1	x_j	x_I	C I	G E	

Z

Intermediate Consumption Matrix

Y

Final Demand Matrix

Total Production by selling industry i = x_i

Total Production by purchasing industry j = x_j

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
		1st *quadrant*			2nd *quadrant*	
Industry 1	z_{11}	…	z_{1I}	y_{11}	… y_{1Q}	x_1
Industry i	…	z_{ij}	…	…	y_{iq} …	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1}	… y_{IQ}	x_I
		3rd *quadrant*			4th *quadrant*	
Imports	…	…	…	…	… …	M
Value added	…	v_{pj}	…	…	y_{pq} …	Y
Total	x_1	x_j	x_I	C I	G E	

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
		1st *quadrant*		2nd *quadrant*		
Industry 1	z_{11}	…	z_{1I}	y_{11}	… y_{1Q}	x_1
Industry i	…	z_{ij}	…	… y_{iq}	…	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1}	… y_{IQ}	x_I
		3rd *quadrant*		4th *quadrant*		
Imports	…	…	…	…	…	M
Value added	…	v_{pj}	…	…	y_{pq} …	Y
Total	x_1	x_j	x_I	C I G	E	

Y

Final Demand Matrix

Macroeconomic Totals

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
		1st *quadrant*			2nd *quadrant*	
Industry 1	z_{11}	…	z_{1I}	y_{11}	… y_{1Q}	x_1
Industry i	…	z_{ij}	…	…	y_{iq} …	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1}	… y_{IQ}	x_I
		3rd *quadrant*			4th *quadrant*	
Imports	…	…	…	…	… …	M
Value added	…	v_{pj}	…	…	y_{pq} …	Y
Total	x_1	x_j	x_I	C I	G E	

V

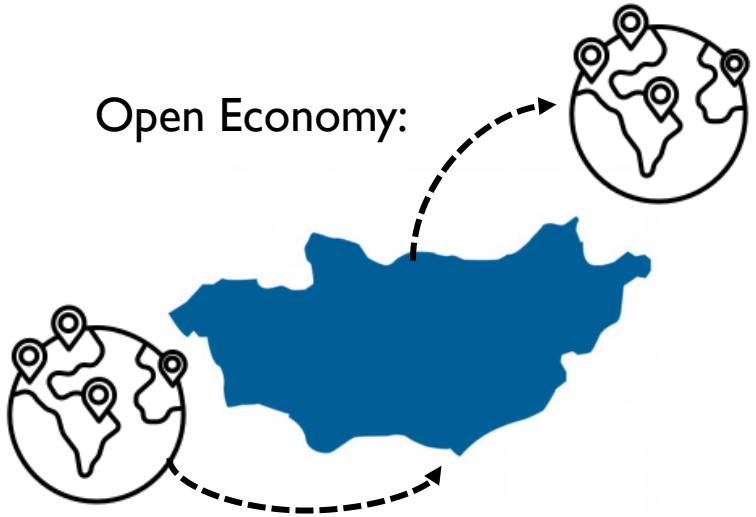
Value Added / Primary Inputs

Open and Closed Economy IO Tables

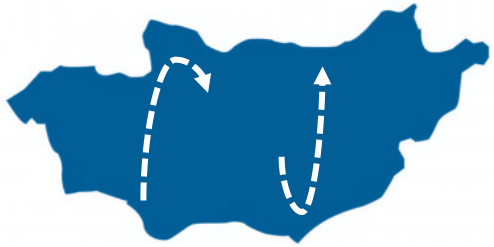
Leontief Output Model

Open Vs. Closed Economy IO Tables

Open Economy:



Closed Economy:



	Industry 1	Industry j	Industry I	Local final demand				Total
		1st *quadrant*			2nd *quadrant*			
Industry 1	z_{11}	…	z_{1I}	y_{11}	…			x_1
Industry i	…	z_{ij}	…	…	y_{iq}			x_i
Industry I	z_{I1}	…	z_{II}	y_{I1}	…			x_I
		3rd *quadrant*			4th *quadrant*			
Value added	…	v_{pj}	…	…	y_{pq}			Y
Total	x_1	x_j	x_I	C	I	G		

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
		1st *quadrant*		2nd *quadrant*		
Industry 1	z_{11}	…	z_{1I}	y_{11} …	y_{1Q}	x_1
				Y		
Industry i	…	z_{ij}	…	… **Final Demand Matrix** y_{iq}	…	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1} …	y_{IQ}	x_I
		3rd *quadrant*		4th *quadrant*		
Imports	…	…	…	… …	…	M
		V				
Value added	…	**Value Added / Primary Inputs** v_{pj}	…	… y_{pq}	…	Y
Total	x_1	x_j	x_I	C I G	E	

Single-region Input-Output Table

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
		1st *quadrant*			2nd *quadrant*	
Industry 1	z_{11}	…	z_{1I}	y_{11}	… y_{1Q}	x_1
Industry i	…	z_{ij}	…	…	y_{iq} …	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1}	… y_{IQ}	x_I
		3rd *quadrant*			4th *quadrant*	
Imports	…	…	…	…	… …	M
Value added	…	v_{pj}	…	…	y_{pq} …	Y
Total	x_1	x_j	x_I	C I	G E	

Single-region Input-Output Table

$$C + I + G + E = M + Y$$

$$C + I + G + E - M = Y$$

$$\boldsymbol{Y = C + I + G + E - M}$$

Closed Economy:

Gross Domestic Product (Y) = Consumption (C) + Investment (I) + Government Expenditure (G)

Open Economy:

Gross Domestic Product (Y) = Consumption (C) + Investment (I) + Government Expenditure (G) + Exports (E) - Imports (M)

Open Vs. Closed Input-Output Table

Intermediate inputs from local industries to local industries

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
	1st quadrant			2nd quadrant		
Industry 1	z_{11}	…	z_{1I}	y_{11}	…	y_{1Q}	x_1
Industry i	…	z_{ij}	…	…	y_{iq}	…	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1}	…	y_{IQ}	x_I
	3rd quadrant			4th quadrant		
Imports	…	…	…	…	…	…	M
Value added	…	v_{pj}	…	…	y_{pq}	…	Y
Total	x_1	x_j	x_I	C	I G	E	

C: Sales of final output for local demand (consumption and investment)

O: Sales of final output for local demand (consumption and investment) + Intermediate and final output for export to RoW

C: Primary inputs (labor, capital, rent)

O: Primary inputs (labor, capital rent) + Imports of intermediate inputs from the Rest of the World (RoW)

C: Primary inputs by final demand category

O: Primary inputs by final demand category + Imported goods for final use

Quick Quiz!

1. In the IO table, which quadrant DOES NOT change while comparing a closed economy IOT to an open economy IOT?

a. 1st Quadrant
b. 2nd Quadrant
c. 3rd Quadrant
d. 4th Quadrant

Quick Quiz!

1. In the IO table, which quadrant DOES NOT change while comparing a closed economy IOT to an open economy IOT?

a. 1st Quadrant
b. 2nd Quadrant
c. 3rd Quadrant
d. 4th Quadrant

Possibilities for descriptive research

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
		1st *quadrant*			2nd *quadrant*	
Industry 1	z_{11}	…		…	y_{1Q}	x_1
Industry i	…	z_{ij}	…	…	y_{iq}	…	x_i
Industry I	z_{I1}	…	z_{II}	y_{I1}	…	y_{IQ}	x_I
		3rd *quadrant*			4th *quadrant*	
Imports	…	…	…	…	…	…	M
Value added	…	v_{pj}	…		…	Y
Total	x_1	x_j	x_I	C I G	E	

z_{ij}/x_j **sales structure**

v_{pj}/Y **contributions to GDP**

Possibilities for descriptive research

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
		1st *quadrant*			2nd *quadrant*	
Industry 1	z_{11}	…	z_{1I}	y_{11}	… y_{1Q}	x_1
Industry i	…	z_{ij}	…	…	y_{iq} …	x_i
Industry I		..			?	x_I
		3rd *quadrant*			4th *quadrant*	
Imports	…	…	…	…	… …	M
Value added	…	v_{pj}	…	…	y_{pq} …	Y
Total	x_1	x_j	x_I	C	I G E	

z_{ij}/x_i **cost structures**

$y_{iq}/C\ (or\ I,G)$ **purchase structures**

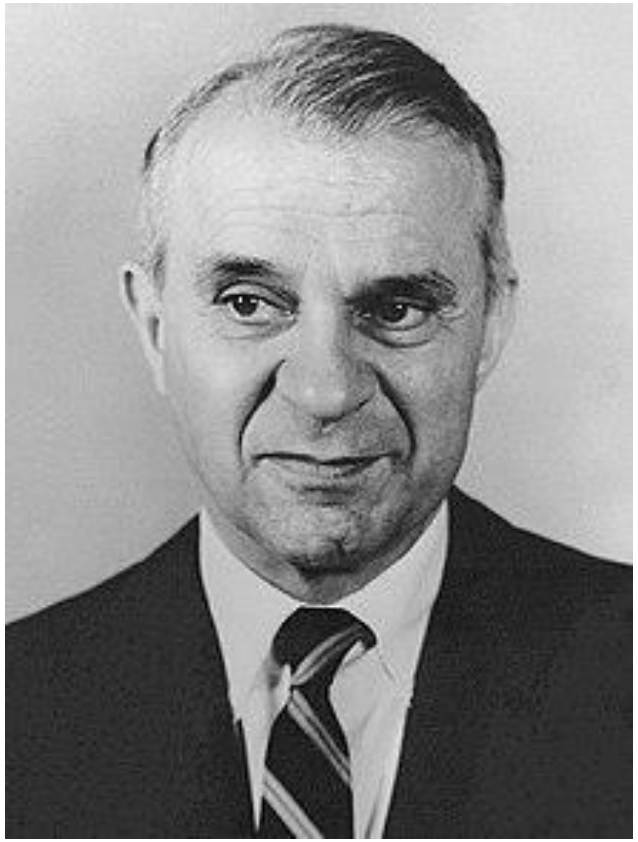
Open and Closed Economy IO Tables

Mathematics of the Closed Economy IO Model

Mathematics of the Closed Economy IO Model

- The main use of IO tables is to provide the data to build IO models; the first of which was formulated by Wassily Leontief (1941), who won the 1972 Nobel Prize in economics for the development of IO analysis.

- The economics of the basic, closed economy IO model, without imports and exports, may be complicated, and its mathematics is simple. It consists of only two equations.



Mathematics of the Closed Economy IO Model

Consists of two simple equations:

EQUATION 1:

Production by industry *i* is determined by the demand for its outputs: Intermediate Output (Quadrant 1) and Final Output (Quadrant 2)

$$x_i = \sum_j z_{ij} + \sum_q y_{iq}, \forall i$$

$$\mathbf{x} = \mathbf{Zi} + \mathbf{Yi} \text{ or } \mathbf{x} = \mathbf{Zi} + \mathbf{y} \qquad (1)$$

Mathematics of the Closed Economy IO Model

Consists of two simple equations:

EQUATION 2:

Demand for intermediate inputs (Q1) and primary inputs (Q3) is proportionally dependent on the size of total output

$$z_{ij} = a_{ij} x_j, \forall i, j$$

$$v_{pj} = c_{pj} x_j, \forall p, j$$

$$\mathbf{Zi} = \mathbf{A}\,\mathbf{x} \qquad (2)$$

$$\mathbf{Vi} = \mathbf{C}\,\mathbf{x} \qquad (3)$$

$\boldsymbol{a_{ij}}$ is the intermediate input coefficient

$\boldsymbol{c_{pj}}$ is the primary input coefficient

Deriving input coefficients: Intermediate Inputs

	Industry 1	Industry j	Industry I
		1st *quadrant*	
Industry 1	z_{11}	…	z_{1I}
Industry i	…	z_{ij}	…
Industry I	z_{I1}	…	z_{II}
		3rd *quadrant*	
Imports	…	…	…
Value added	…	v_{pj}	…
Total	x_1	x_j	x_I

Intermediate Consumption Matrix Z

Total Production by purchasing industry j = x_j

$$a_{ij} = \frac{z_{ij}}{x_j} \qquad \mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$$

Share of each intermediate input a_{ij} to total production

Ex: $a_{I1} = \frac{z_{I1}}{x_1}, a_{1I} = \frac{z_{1I}}{x_I}$

Deriving input coefficients: Primary Inputs

	Industry 1	Industry j	Industry I
		1st *quadrant*	
Industry 1	z_{11}	…	z_{1I}
Industry i	…	z_{ij}	…
Industry I	z_{I1}	…	z_{II}
		3rd *quadrant*	
		Value Added / Primary Inputs	
Imports		…	…
Value added	…	v_{pj}	…
Total	x_1	x_j	x_I

Total Production by purchasing industry j = x_j

$$c_{pj} = \frac{v_{pj}}{x_j} \qquad \mathbf{C} = \mathbf{V}\hat{\mathbf{x}}^{-1}$$

Share of each primary input v_{pj} to total production

Ex: $c_{P1} = \frac{v_{P1}}{x_1}$, $c_{1I} = \frac{v_{1P}}{x_I}$

v_{pj} may represent employment, CO2 emissions, or energy use of industry *j*

c_{pj} may represent the employment, CO_2 emissions, or energy use of industry j ***per unit of output of industry j***

Deriving input coefficients

	Industry 1	Industry j	Industry I
	Intermediate Input Coeff. Matrix		
Industry 1	a_{11}	…	a_{1I}
Industry i	…	a_{ij}	…
Industry I	a_{I1}	…	a_{II}
	Primary Input Coeff. Matrix		
Imports	…	…	…
Value added	…	c_{pj}	…
Total	1	1	1

Formula	Matrix Notation
$a_{ij} = \frac{z_{ij}}{x_j}$	$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$
$c_{pj} = \frac{v_{pj}}{x_j}$	$\mathbf{C} = \mathbf{V}\hat{\mathbf{x}}^{-1}$

$$\mathbf{i}'\mathbf{A} + \mathbf{i}'\mathbf{C} = \mathbf{i}'$$

Mathematical Solution of the IO Model: **x**

Recall our first two equations:

$$\mathbf{x} = \mathbf{Zi} + \mathbf{y} \qquad (1)$$

$$\mathbf{Zi} = \mathbf{Ax} \qquad (2)$$

$$\mathbf{x} = \mathbf{Ax} + \mathbf{y}$$

$$\mathbf{x} - \mathbf{Ax} = \mathbf{y}$$

$$(\mathbf{I} - \mathbf{A})\mathbf{x} = \mathbf{y}$$

$$(\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A})^{-1}\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$$

Leontief Inverse:

$$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$$

Solution for total output by the industry: $x_i \in \mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} = \mathbf{Ly} \qquad (4)$

Mathematical Solution of the IO Model: **Z** and **V**

Recall:

$$\mathbf{x} = \mathbf{Ly} \qquad (4)$$

$$\mathbf{Zi} = \mathbf{Ax} \qquad (2)$$

$$\mathbf{Vi} = \mathbf{Cx} \qquad (3)$$

Solutions for the matrices with intermediate and primary inputs:

$$z_{ij} \in \mathbf{Z} = \mathbf{AL}\hat{\mathbf{y}} \qquad (5)$$

$$v_{pj} \in \mathbf{V} = \mathbf{CL}\hat{\mathbf{y}} \qquad (6)$$

A note: closed vs. open economy

	Closed Economy	Open Economy
Intermediate Input Coefficients	Technical Coefficients	Technical Coefficients x Trade Coefficients

	Industry 1	Industry j	Industry I
	Technical Coefficients Matrix		
Industry 1	a_{11}	…	a_{1I}
Industry i	…	a_{ij}	…
Industry I	a_{I1}	…	a_{II}

Industry 1	Industry j	Industry I
Tech. Coeff x Trade Coeff.		
$a_{11}^{rr} = m_{11}^{rr} a_{11}^{\cdot r}$	…	$a_{1I}^{rr} = m_{1I}^{rr} a_{1I}^{\cdot r}$
…	$a_{ij}^{rr} = m_{ij}^{rr} a_{ij}^{\cdot r}$	…
$a_{I1}^{rr} = m_{I1}^{rr} a_{I1}^{\cdot r}$	…	$a_{II}^{rr} = m_{II}^{rr} a_{II}^{\cdot r}$

Quick Quiz!

2. Which matrix represents the "technical coefficients" or the "intermediate inputs coefficients" matrix?

a. **A** matrix
b. **Z** matrix
c. **C** matrix
d. **x** matrix

Quick Quiz!

2. Which matrix represents the "technical coefficients" or the "intermediate inputs coefficients" matrix?

a. **A** matrix
b. **Z** matrix
c. **C** matrix
d. **x** matrix

Open and Closed Economy IO Tables

Economics of the Closed Economy IO Model

Economics of the Closed Economy IO Model

Recall:

$$\boxed{\mathbf{Z}} = \boxed{\mathbf{AL}}\ \boxed{\hat{\mathbf{y}}} \qquad (5)$$

$$\boxed{\mathbf{V}} = \boxed{\mathbf{CL}}\ \boxed{\hat{\mathbf{y}}} \qquad (6)$$

These equations have the general structure of the solution of any model:

Endogenous variable A =

model X's A multiplier of B * **exogenous variable B**

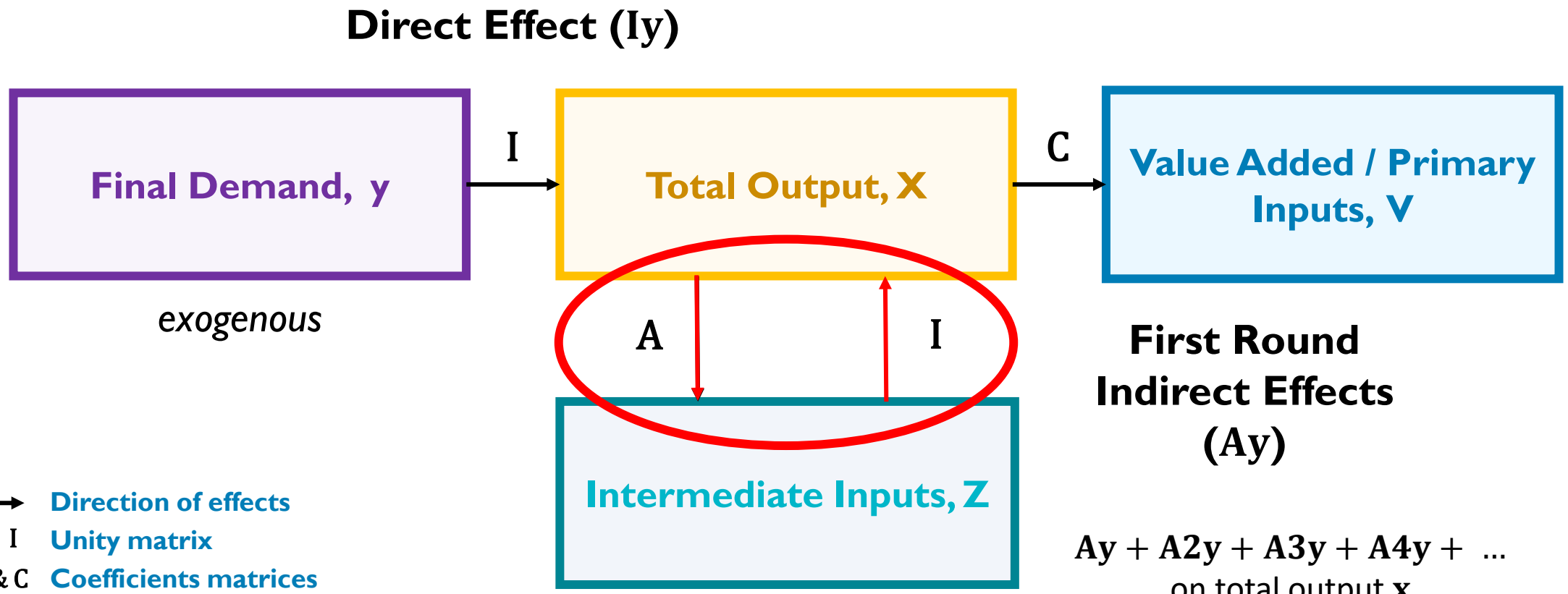
Endogenous variables: $\boldsymbol{Z}$ and $\boldsymbol{V}$

Exogenous variable: $\hat{\mathbf{y}}$

Multipliers: $\mathbf{AL}$ and $\mathbf{CL}$

Economic Causality of the Basic IO Model

How are these variable interrelated?



$\mathbf{Ay} + \mathbf{A2y} + \mathbf{A3y} + \mathbf{A4y} + \ \ldots$

on total output $\mathbf{x}$

→ Direction of effects

I Unity matrix

A & C Coefficients matrices

Economic Causality of the Basic IO Model

$$\mathbf{x} = \mathbf{Ay} + \mathbf{A2y} + \mathbf{A3y} + \mathbf{A4y} + \ \ldots$$

$$\mathbf{x} = (\mathbf{A} + \mathbf{A2} + \mathbf{A3} + \mathbf{A4} + \ \ldots)\mathbf{y}$$

$$(1 + a + a^2 + a^3 + \cdots)$$

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$$

$$\mathbf{x} = \mathbf{Ly} \qquad (10)$$

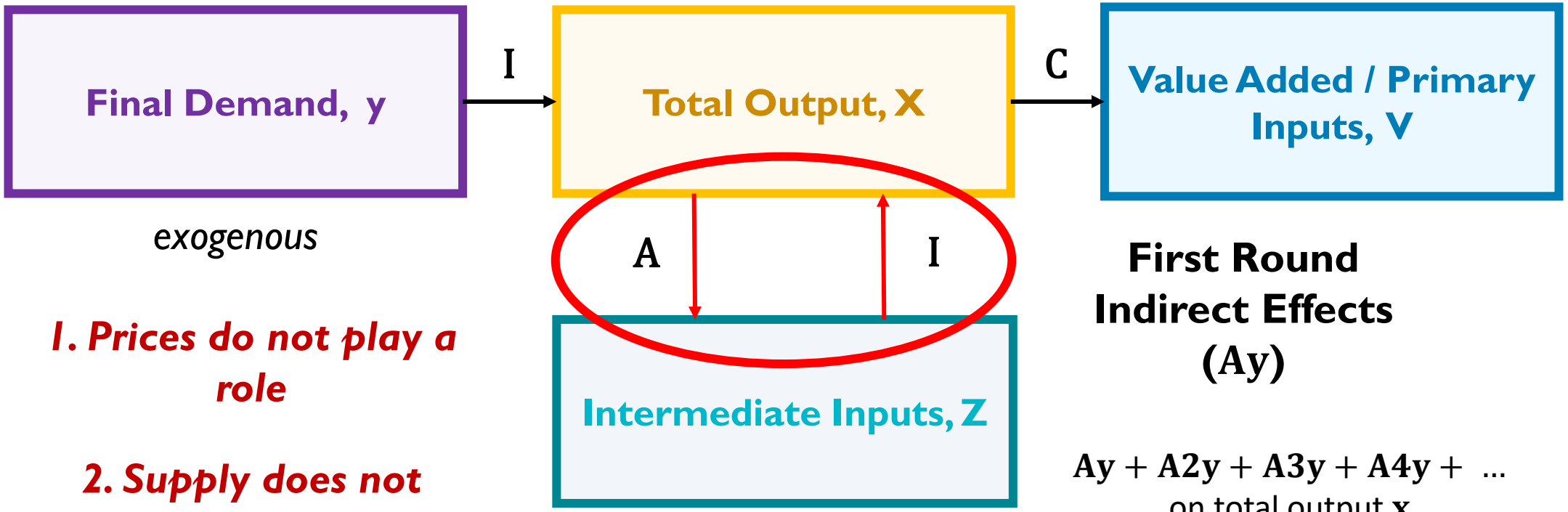
Recall: $\mathbf{i}'\mathbf{A} + \mathbf{i}'\mathbf{C} = \mathbf{i}'$

Sufficient conditions:
All column sums of $\mathbf{A}$ < 1
All column sums of primary input matrix > 0

Economic Causality of the Basic IO Model

"Demand-Driven IO Quantity Model"

Direct Effect (Iy)



exogenous

1. Prices do not play a role

2. Supply does not play an active role

First Round Indirect Effects (Ay)

$\mathbf{Ay + A2y + A3y + A4y + \ \ldots}$
on total output $\mathbf{x}$

“Demand-Driven IO Model”

Demand is met without any restriction on the supply side, i.e. there are no capacity constraints nor shortages of any kind.

- Explicit assumption: **Demand is always fully met**
- Implicit assumption: supply of inputs (primary and intermediate) is **perfectly price elastic.**

An important implication:

The IO model will produce an overestimation of the production and employment effects of any increase in final demand whenever an economy is close to the top of its business cycle.

Quick Quiz!

3. Which of the following is NOT an assumption or implication of the "demand driven IO model"

a. Demand is met without any restriction on the supply side
b. Supply of inputs is perfectly price elastic
c. The size of final demand or y is exogenous
d. The size of output or $\mathbf{x}$ is exogenous

Quick Quiz!

3. Which of the following is NOT an assumption or implication of the "demand driven IO model"

a. Demand is met without any restriction on the supply side
b. Supply of inputs is perfectly price elastic
c. The size of final demand or y is exogenous
d. The size of output or $\mathbf{x}$ is exogenous

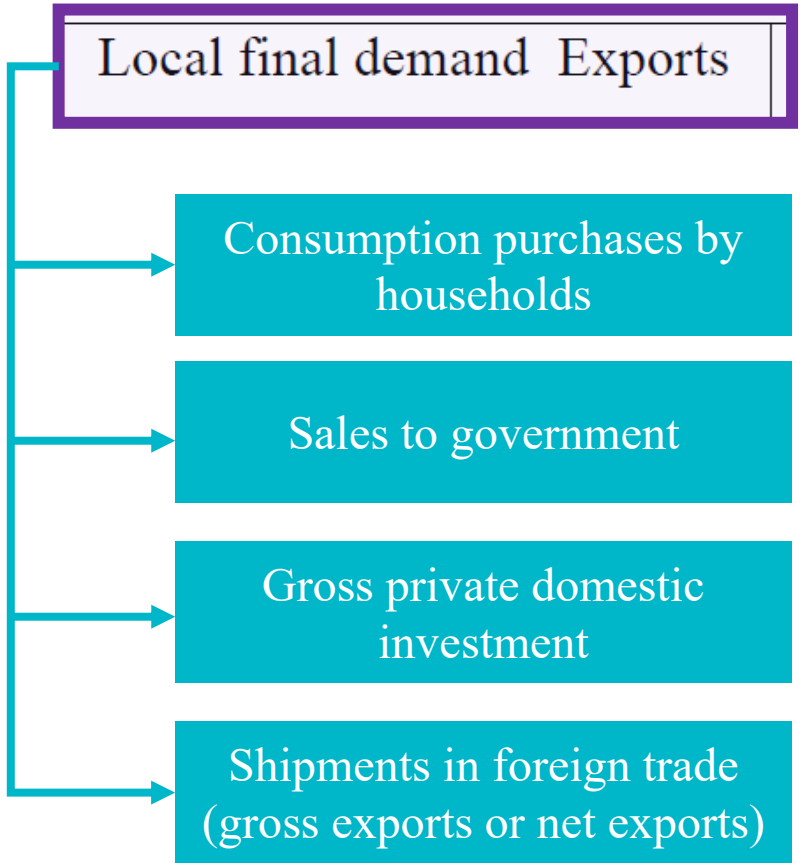
Extension of the Leontief Output Model

Closing the model with respect to Households

Closing the Model with respect to Households

	Industry 1	Industry j	Industry I	Local final demand	Exports	Total
	1st quadrant			2nd quadrant		
Industry 1	z_{11}	…	z_{1I}	y_{11}	… y_{1Q}	x_1
	Z			Y		
Industry i	…	z_{ij}	…	…	y_{iq} …	x_i
	Intermediate Consumption Matrix			Final Demand Matrix		
Industry I	z_{I1}	…	z_{II}	y_{I1}	… y_{IQ}	x_I
	3rd quadrant			4th quadrant		
	V					
Imports	…	…	…	…	… …	M
Value added	…	v_{pj}	…	…	y_{pq} …	Y
	Value Added / Primary Inputs					
Total	x_1	x_j	x_I	C I	G E	

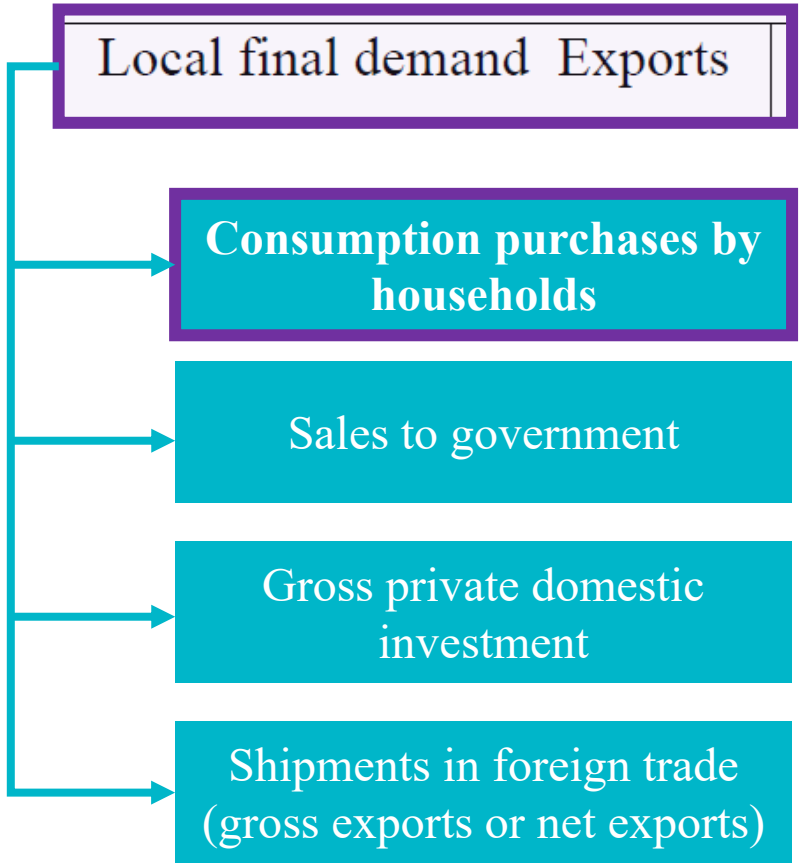
Closing the Model with respect to Households



Closing the Model with respect to Households

"Exogenous" categorization of Households is a strain on basic economic theory. Why?

- **Households** (consumers) earn incomes as payment for their labor inputs to production processes → spend their income in patterned ways
- Increase in labor inputs needed due to increased output → increase in HH's consumption



Closing the Model with respect to Households

		Buying Sector				
		1	$\cdots$	j	$\cdots$	n
Selling Sector	1	z_{11}	$\cdots$	z_{1j}	$\cdots$	z_{1n}
	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	i	z_{i1}	$\cdots$	z_{ij}	$\cdots$	z_{in}
	$\vdots$	$\vdots$		$\vdots$		$\vdots$
	n	z_{n1}	$\cdots$	z_{nj}	$\cdots$	z_{nn}

Closing the Model with respect to Households

		Buying Sector					
		1	$\cdots$	j	$\cdots$	n	*Households (Consumers)*
Selling Sector	1	z_{11}	$\cdots$	z_{1j}	$\cdots$	z_{1n}	$z_{1,n+1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	i	z_{i1}	$\cdots$	z_{ij}	$\cdots$	z_{in}	$z_{i,n+1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	n	z_{n1}	$\cdots$	z_{nj}	$\cdots$	z_{nn}	$z_{n,n+1}$
	Households (Labor)	$z_{n+1,1}$	$\cdots$	$z_{n+1,j}$	$\cdots$	$z_{n+1,n}$	$z_{n+1,n+1}$

Row showing the distribution of the HH sector's output (labor services) among the various sectors

Column showing the structure of the HH sector's purchases (consumption) distributed among the sectors

Closing the model with respect to Households

Closing the Model with respect to Households

		Buying Sector					
		1	$\cdots$	j	$\cdots$	n	*Households (Consumers)*
Selling Sector	1	z_{11}	$\cdots$	z_{1j}	$\cdots$	z_{1n}	$z_{1,n+1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	i	z_{i1}	$\cdots$	z_{ij}	$\cdots$	z_{in}	$z_{i,n+1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	n	z_{n1}	$\cdots$	z_{nj}	$\cdots$	z_{nn}	$z_{n,n+1}$
	Households (Labor)	$z_{n+1,1}$	$\cdots$	$z_{n+1,j}$	$\cdots$	$z_{n+1,n}$	$z_{n+1,n+1}$

Dollar flows ***from*** consumers, representing values of household purchases of the goods of the *n* sectors

Household purchases of labor services

Dollar flows ***to*** consumers (as wages and salaries received by households) from *n* sectors

Closing the Model with respect to Households

Recall:

$$x_i = z_{i1} + z_{i2} + \cdots + z_{in} + f_i$$

Production by industry *i* is determined by the demand for its outputs: Intermediate Output (Quadrant 1) and Final Output (Quadrant 2)

Closing the model with respect to households modifies this equation to:

$$x_i = z_{i1} + z_{i2} + \cdots + z_{in} + z_{i,n+1} + f_i^*$$

$\mathbf{f}^*$ represents the remaining final demand for sector i output

Households' final demand

New equation for total "output" of the Household sector:

$$x_{n+1} = z_{n+1,1} + z_{n+1,2} + \cdots + z_{n+1,n} + z_{n+1,n+1} + f_{n+1}^*$$

Quick Quiz!

4. In the given closed IO table wrt HHs, what does the highlighted term represent?

		Buying Sector					
		1	$\cdots$	j	$\cdots$	n	*Households (Consumers)*
Selling Sector	1	z_{11}	$\cdots$	z_{1j}	$\cdots$	z_{1n}	$z_{1,n+1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	i	z_{i1}	$\cdots$	z_{ij}	$\cdots$	z_{in}	$z_{i,n+1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	n	z_{n1}	$\cdots$	z_{nj}	$\cdots$	z_{nn}	$z_{n,n+1}$
	Households (Labor)	$z_{n+1,1}$	$\cdots$	$z_{n+1,j}$	$\cdots$	$z_{n+1,n}$	$\boxed{z_{n+1,n+1}}$

$$\boxed{z_{n+1,n+1}}$$

a. Wages of Households

b. Household purchases of labor services

c. Household purchases of goods

Quick Quiz!

4. In the given closed IO table wrt HHs, what does the highlighted term represent?

		Buying Sector					
		1	$\cdots$	j	$\cdots$	n	*Households (Consumers)*
Selling Sector	1	z_{11}	$\cdots$	z_{1j}	$\cdots$	z_{1n}	$z_{1,n+1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	i	z_{i1}	$\cdots$	z_{ij}	$\cdots$	z_{in}	$z_{i,n+1}$
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
	n	z_{n1}	$\cdots$	z_{nj}	$\cdots$	z_{nn}	$z_{n,n+1}$
	Households (Labor)	$z_{n+1,1}$	$\cdots$	$z_{n+1,j}$	$\cdots$	$z_{n+1,n}$	$\boxed{z_{n+1,n+1}}$

$$\boxed{z_{n+1,n+1}}$$

a. Wages of Households

b. Household purchases of labor services

c. Household purchases of goods

Closing the Model with respect to Households – HH Input Coefficients

Household input coefficients are found in the same manner as any other element in an input-output coefficients table:

$$a_{n+1,j} = z_{n+1,j}/x_j$$

representing the value of household services (labor) used per dollars' worth of j's output.

Household consumption coefficients are given by:

$$a_{i,n+1} = z_{i,n+1,}/x_{n+1}$$

Drawback? "Frozen" household behavior.

Closing the Model with respect to Households – HH Input Coefficients

Given $\underbrace{x_i = z_{i1} + z_{i2} + \cdots + z_{in} + z_{i,n+1} + f_i^*}_{\text{Production by industry for an endogenized Households model}}$ and $\underbrace{a_{n+1,j} = z_{n+1,j}/x_j}_{\text{HH Input Coefficients}}$

We also have, for the added equation:

$$x_{n+1} = a_{n+1,1}x_1 + a_{n+1,2}x_2 + \cdots + a_{n+1,n}x_n + a_{n+1,n+1}x_{n+1} + f_{n+1}^*$$

Rewriting both equations:

$$-a_{i1}x_1 - a_{i2}x_2 - \cdots + (1 - a_{ii})x_i - \cdots - a_{in}x_n - a_{i,n+1}x_{n+1} = f_i^*$$

$$-a_{n+1,1}x_1 - a_{n+1,2}x_2 - \cdots - a_{n+1,n}x_n + (1 - a_{n+1,n+1})x_{n+1} = f_{n+1}^*$$

Closing the Model with respect to Households – HH Input Coefficients

Let

$$\mathbf{h}_R = (a_{n+1,1} \quad a_{n+1,2} \quad \cdots \quad a_{n+1,n}),$$

row vector of labor input coefficients

$$\mathbf{h}_C = \begin{bmatrix} a_{1,n+1} \\ \vdots \\ a_{n,n+1} \end{bmatrix}$$

column vector of household consumption coefficients

$$h = a_{n+1,n+1}$$

$\overline{\mathbf{A}}$ be the (n+1) by (n+1) technical coefficients matrix with households included

$$\overline{\mathbf{A}} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & a_{1,n+,} \\ a_{21} & a_{22} & \cdots & a_{2n} & a_{2,n+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & a_{n,n+1} \\ a_{n+1,1} & a_{n+1,2} & \cdots & a_{n+1,n} & a_{n+1,n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{h}_c \\ \mathbf{h}_R & h \end{bmatrix}$$

Closing the Model with respect to Households – HH Input Coefficients

Let

$\bar{\mathbf{x}}$ be the (n+1)-element column vector of gross outputs, $\mathbf{f}^*$ be the n-element vector of remaining final demands for output of the original n sectors, $\bar{\mathbf{f}}$ be the (n+1)-element vector of final demands, including that for the output of households:

$$\bar{\mathbf{x}} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x_{n+1} \end{pmatrix} = \begin{pmatrix} \mathbf{x} \\ x_{n+1} \end{pmatrix}, \quad \bar{\mathbf{f}} = \begin{pmatrix} f_1^* \\ f_2^* \\ \vdots \\ f_n^* \\ f_{n+1}^* \end{pmatrix} = \begin{pmatrix} \mathbf{f}^* \\ f_{n+1}^* \end{pmatrix}$$

Closing the Model with respect to Households – HH Input Coefficients

The new system of n+1 equations, with households endogenous, can be represented as:

$$\boxed{(\mathbf{I}-\overline{\mathbf{A}})\overline{\mathbf{x}}=\overline{\mathbf{f}}}$$

$$\left\{\begin{bmatrix}1 & 0 & 0 & \dots & 0\\ 0 & 1 & 0 & \dots & 0\\ 0 & 0 & 1 & \dots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \dots & 1\end{bmatrix}-\begin{bmatrix}a_{11} & a_{12} & \dots & a_{1n} & a_{1,n+,}\\ a_{21} & a_{22} & \dots & a_{2n} & a_{2,n+1}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ a_{n1} & a_{n2} & \dots & a_{nn} & a_{n,n+1}\\ a_{n+1,1} & a_{n+1,2} & \dots & a_{n+1,n} & a_{n+1,n+1}\end{bmatrix}\right\}\begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_n\\ x_{n+1}\end{pmatrix}=\begin{pmatrix}f_1^*\\ f_2^*\\ \vdots\\ f_n^*\\ f_{n+1}^*\end{pmatrix}$$

$$\begin{bmatrix}1-a_{11} & -a_{12} & \dots & -a_{1n} & -a_{1,n+,}\\ -a_{21} & 1-a_{22} & \dots & -a_{2n} & -a_{2,n+1}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ -a_{n1} & -a_{n2} & \dots & 1-a_{nn} & -a_{n,n+1}\\ -a_{n+1,1} & -a_{n+1,2} & \dots & -a_{n+1,n} & 1-a_{n+1,n+1}\end{bmatrix}\begin{pmatrix}x_1\\ x_2\\ \vdots\\ x_n\\ x_{n+1}\end{pmatrix}=\begin{pmatrix}f_1^*\\ f_2^*\\ \vdots\\ f_n^*\\ f_{n+1}^*\end{pmatrix}$$

Closing the Model with respect to Households – System of n+1 Equations

The new system of n+1 equations, with households endogenous, can be represented as:

$$(\mathbf{I} - \overline{\mathbf{A}})\overline{\mathbf{x}} = \overline{\mathbf{f}}$$

$$\begin{bmatrix} \mathbf{I} - \mathbf{A} & -\mathbf{h}_C \\ -\mathbf{h}_R & 1 - h \end{bmatrix} \begin{pmatrix} \mathbf{x} \\ x_{n+1} \end{pmatrix} = \begin{pmatrix} \mathbf{f}^* \\ f^*_{n+1} \end{pmatrix}$$

Given that $(\mathbf{I} - \overline{\mathbf{A}})$ is nonsingular:

$$\begin{pmatrix} \mathbf{x} \\ x_{n+1} \end{pmatrix} = \begin{bmatrix} \mathbf{I} - \mathbf{A} & -\mathbf{h}_C \\ -\mathbf{h}_R & 1 - h \end{bmatrix}^{-1} \begin{pmatrix} \mathbf{f}^* \\ f^*_{n+1} \end{pmatrix}$$

$$\boxed{\overline{\mathbf{x}} = (\mathbf{I} - \overline{\mathbf{A}})^{-1}\overline{\mathbf{f}} = \overline{\mathbf{L}}\overline{\mathbf{f}}}$$

References

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- Oosterhaven, J. (2019). Rethinking Input-Output Analysis: A Spatial Perspective

Thank you!