Basic Framework of Input-Output Analysis

ADB-CI Virtual Workshop on Input-Output Analysis

Outline

Supply and Use Table (SUT) to Input-Output Table (IOT) Transformation

Structure of Input-Output Tables

Supply and Use Table (SUT) to Input-Output Table (IOT) Transformation

Discussion outline

Background

- ✓ What are SUTs and IOTs?
- ✓ Why transform SUT into an IOT?
- ✓ Types of IOT
- ✓ Requirements for Transformation

Models of Transformation

- ✓ Model Assumptions
- ✓ Choice of IO Model
- ✓ SUT-IOT Transformation using Georgia dataset (worksheet)

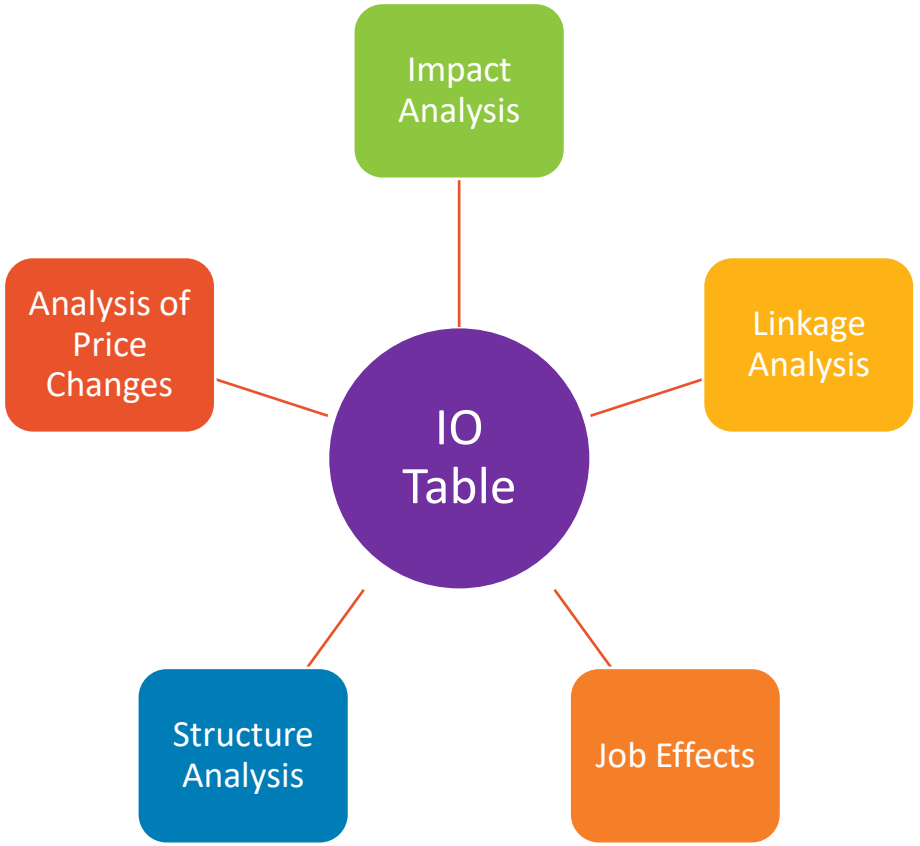
What are SUT and IOT? Why Transform SUT into IOT?

SUTs provide a **statistical framework** to

- ensure **coherence** and **consistency** in various parts of the National accounts
- infer detailed information on the **production processes**, the **interdependencies in production**, the **use of goods and services**, and **generation of income**

However, **Imposing assumptions** about inputs and outputs on SUTs are often necessary to **improve** its **analytical credence**.

Such assumptions can be **imposed by transforming the SUT into symmetric IOTs.**



SUT and IOT

SUTs are the building blocks of IOTs

IOTs are necessarily square but SUTs are not.

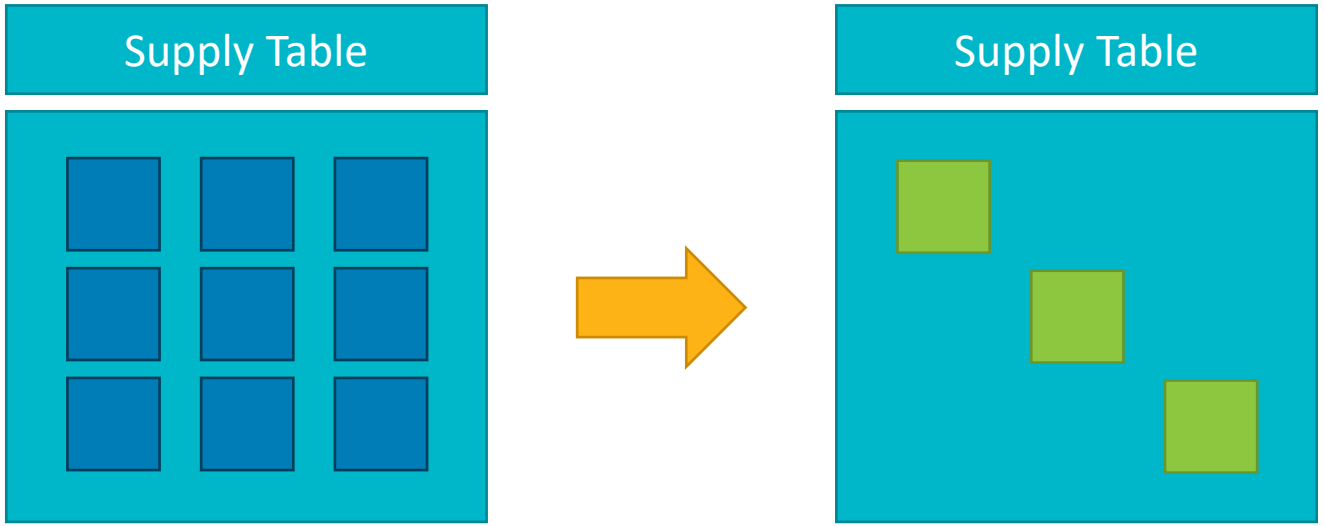
SUTs can be rectangular implying that there are more products or commodities than industries

SUTs are produced for Commodity x Industry based on source data.

IOTs could be either produced as Product-by-Product IOTs and Industry-by-Industry IOTs.

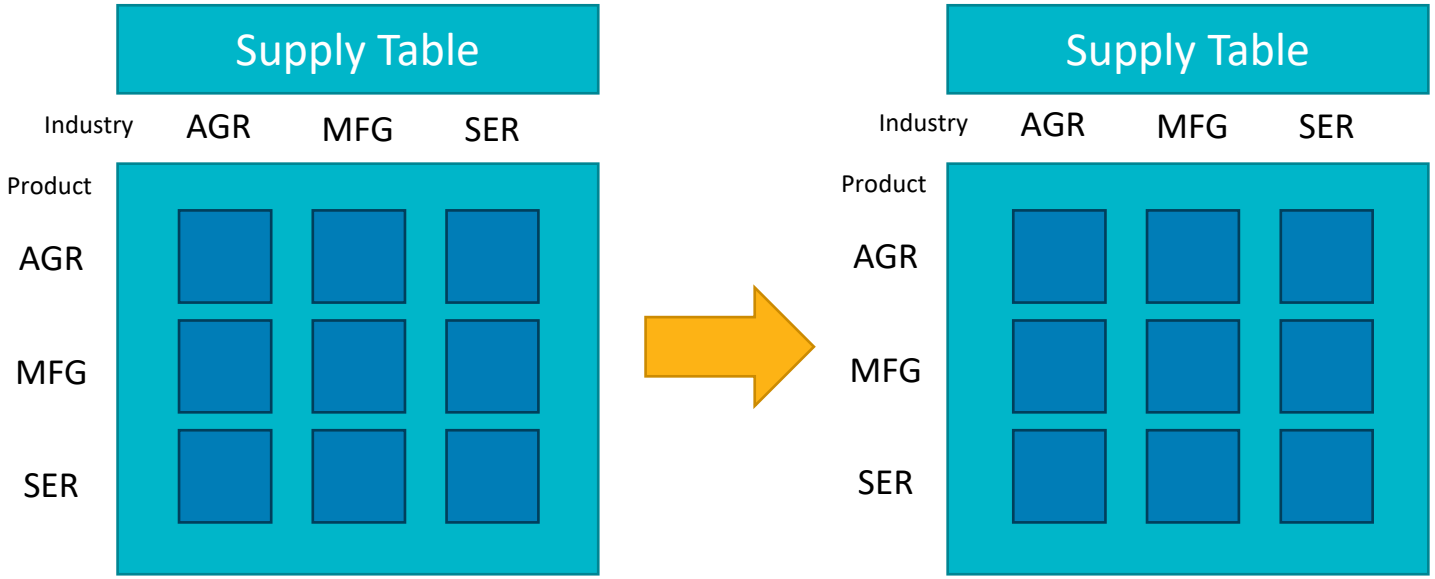
Objective of the SUT-IOT transformation

- The objective of the SUT-IOT transformation is to “diagonalize” the supply table.



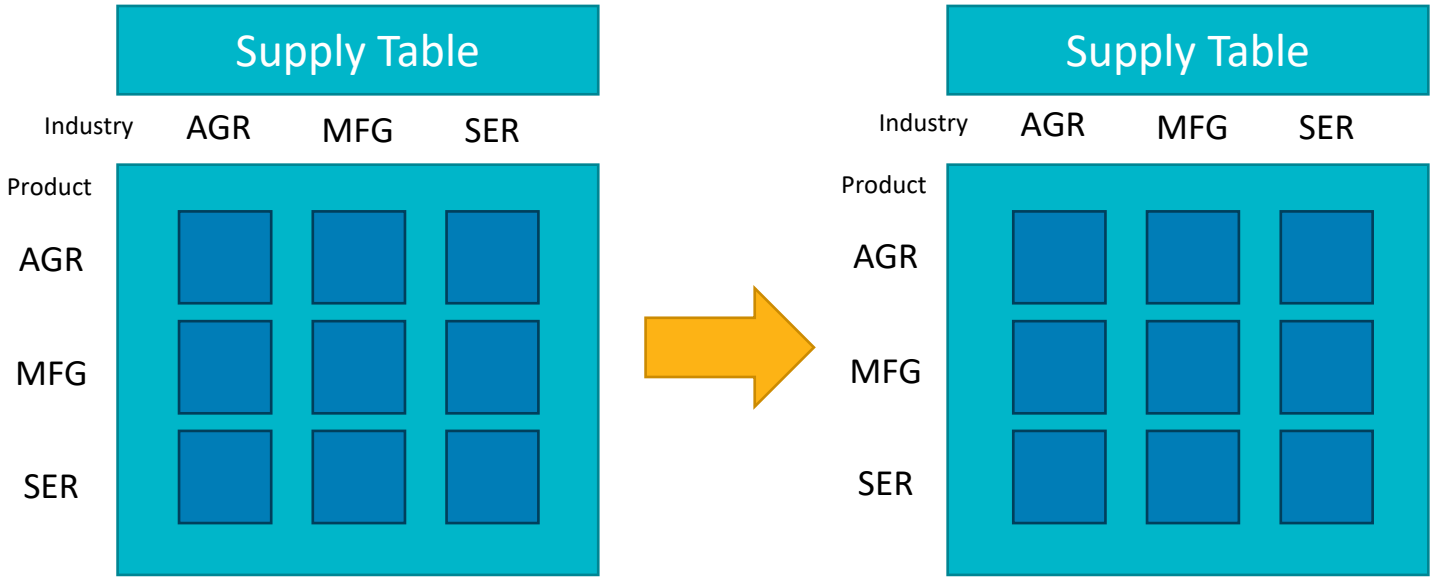
Objective of the SUT-IOT transformation

- The supply table is diagonalized either by:
  - Moving the secondary products (and its related inputs and GVA) of any industry to the industry where these products are primary [This results in a "product-by-product" IOT.], or



Objective of the SUT-IOT transformation

- The supply table is diagonalized either by:
  - Rearranging the products by distributing each product to the industries where it is actually produced [This results in an “industry-by-industry” IOT.].



Types of IOT

Product-by-product IOT

- In a product-by-product IOT, secondary products are moved to the industries where they are primary outputs.
- Demonstrate the technological relations between products and homogenous units of production.

Industry-by-industry IOT

- In an industry-by-industry IOT, rows are redefined such that each row includes a mix of the primary and secondary outputs of each corresponding industry.
- Describe inter-industry relations.

Requirements for Transformation

- Supply Table at Basic Price
- Use Table at Basic Price
- Domestic Use table at Basic Price
- Import use table at Basic Price

UN Recommendation: Separate compilation of IOTs for domestic output and imported products (UN, 2008: para. 12.18, p. 326)

Models of Transformation

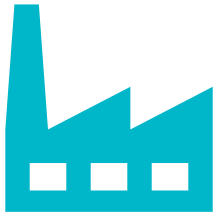
- There are four main models of transformation (A, B, C, and D)
- Models can be grouped according to assumptions

Product-by-product IOT

Technology Assumption

Model A: Product technology

Model B: Industry technology



Industry-by-industry IOT

Sales Structure

Model C: Fixed industry sales structure

Model D: Fixed product sales structure

Models of Transformation

AGR-Agriculture Sector
MFG-Manufacturing
SER- Services
Total IC- Total Intermediate Consumption
FCE-Final Consumption Expenditure
GFCF-Gross Fixed Capital Formation
Total FD- Total Final Demand

Supply Table at Basic Prices

	AGR	MFG	SER	Total Output	Imports	Total Supply
AGR	26	5	7	38	15	**53**
MFG	1	403	40	444	401	**845**
SER	13	26	717	756	145	**901**
Total	**40**	**433**	**765**	**1238**	**561**	**1800**

Domestic Use Table at Basic Prices

	AGR	MFG	SER	Total IC	FCE	GFCF	Exports	Total FD	Total Use
AGR	4	9	1	14	13	0	10	24	**38**
MNF	8	108	46	162	32	74	176	282	**444**
SER	4	63	211	278	345	21	113	478	**756**
Imports	3	124	78	205	75	37	245	357	**561**
GVA	22	130	428	580					**580**
Total	**40**	**433**	**765**	**1,238**	**465**	**131**	**544**	**1,141**	

Source: UN (2018, p. 330)

Models of Transformation

- Model A [Product Technology Assumption]
  - Each product is produced in its own specific way, regardless of the industry where it is produced.
  - Most applicable in cases where technologies of primary and secondary products are independent.
  - Requires a square SUT.
  - Application:
    - Compute for Matrix D [market share matrix] to derive the transformation matrix.
    - Pre-multiply the Domestic Use Table to the Transformation Matrix to obtain the Product-by-Product IOT.

Model A

Supply Table at Basic Prices

	AGR	MFG	SER	Domestic Output
AGR	26	5	7	38
MFG	1	403	40	444
SER	13	26	717	756
Total Output at BP	**40**	**433**	**765**	**1238**

D Matrix: Market Share Matrix

	AGR	MFG	SER
AGR	0.68	0.00	0.02
MFG	0.14	0.91	0.03
SER	0.19	0.09	0.95

For the first row, we have 26/38, 5/38 and 7/38

- Transformation Matrix = $(\mathbf{D'})^{-1}$

  Note: ' (apostrophe) denotes matrix transposition.

Transformation Matrix for Model A

	AGR	MFG	SER
AGR	1.48	- 0.21	- 0.27
MFG	- 0.00	1.11	- 0.11
SER	- 0.03	- 0.04	1.06

Model A

Product-by-Product IO Table (Model A)

	AGR	MFG	SER	Total IC	FCE	GFCF	Exports	Total FD	Output
AGR	6	9	- 2	14	13	0	10	24	**38**
MFG	10	116	36	162	32	74	176	282	**444**
SER	- 0	61	217	278	345	21	113	478	**756**
Imports	1	134	70	205	75	37	245	357	**561**
GVA	20	124	436	580					**580**
Input	**38**	**444**	**756**	**1,238**	**465**	**131**	**544**	**1,141**	

- Entries in the Final Demand Matrix are unaltered from the domestic Use Table.
- The transformation matrix is only pre-multiplied to the IC matrix, Imports vector and the GVA vector.
- Existence of Negative Entries: Drawback of the Model

Model of Transformation

- Model B [Industry Technology Assumption]
  - Each industry has its own specific way of production, irrespective of the product mix.
  - Usually used when several products are produced in a single production process.
  - Applying this model <u>does not</u> require a square SUT.

  - Application:
    - Compute for Matrix C [product mix matrix] to derive the transformation matrix.
    - Pre-multiply the Domestic Use Table to the Transformation Matrix to obtain the Product-by-Product IOT.

Model B

Supply Table at Basic Prices

	AGR	MFG	SER	Domestic Output
AGR	26	5	7	38
MFG	1	403	40	444
SER	13	26	717	756
Total Output at BP	**40**	**433**	**765**	**1238**

C Matrix: Product Mix Matrix

	AGR	MFG	SER
AGR	0.64	0.01	0.01
MFG	0.03	0.93	0.05
SER	0.33	0.06	0.94

For the first column, we have 26/40, 1/40 and 13/40

- Transformation Matrix = **C'**

Transformation Matrix for Model B

	AGR	MFG	SER
AGR	0.64	0.03	0.33
MFG	0.01	0.93	0.06
SER	0.01	0.05	0.94

Model B

Product-by-Product IO Table (Model B)

	AGR	MFG	SER	Total IC	FCE	GFCF	Exports	Total FD	Output
AGR	3	9	3	14	13	0	10	24	**38**
MFG	7	103	52	162	32	74	176	282	**444**
SER	5	70	203	278	345	21	113	478	**756**
Imports	4	119	82	205	75	37	245	357	**561**
GVA	19	144	416	580					**580**
Input	**38**	**444**	**756**	**1,238**	**465**	**131**	**544**	**1,141**	

- Entries in the Final Demand Matrix are unaltered from the domestic Use Table.
- The transformation matrix is only pre-multiplied to the IC matrix, Imports vector and the GVA vector.
- No negatives in the model but the absence of negatives does not imply that results are more plausible.

Models of Transformation

- Model C [Fixed Industry Sales Structure Assumption]
  - Each industry has its own specific sales structure, irrespective of its product mix.
  - Requires a square SUT.

  - Application:
    - Compute for Matrix C [product mix matrix] to derive the transformation matrix.
    - **Post-multiply** the Domestic Use Table to the Transformation Matrix to obtain the Industry-by-Industry IOT.

Model C

Supply Table at Basic Prices

	AGR	MFG	SER	Domestic Output
AGR	26	5	7	38
MFG	1	403	40	444
SER	13	26	717	756
Total Output at BP	**40**	**433**	**765**	**1238**

C Matrix: Product Mix Matrix

	AGR	MFG	SER
AGR	0.64	0.01	0.01
MFG	0.03	0.93	0.05
SER	0.33	0.06	0.94

For the first column, we have 26/40, 1/40 and 13/40

- Transformation Matrix = $\mathbf{C}^{-1}$

Transformation Matrix for Model C

	AGR	MFG	SER
AGR	1.58	- 0.02	- 0.01
MFG	- 0.03	1.08	- 0.06
SER	- 0.55	- 0.06	1.07

Model C

Industry-by-Industry IO Table (Model C)

	AGR	MFG	SER	Total IC	FCE	GFCF	Exports	Total FD	Output
AGR	7	12	- 3	15	15	- 2	12	25	**40**
MFG	8	112	37	158	13	79	183	275	**433**
SER	1	56	224	281	361	18	105	484	**765**
Imports	3	124	78	205	75	37	245	357	**561**
GVA	22	130	428	580					**580**
Input	**40**	**433**	**765**	**1,238**	**465**	**131**	**544**	**1,141**	

- Entries in the Imports and GVA Vectors are unaltered from the Domestic Use Table.
- The transformation matrix is only pre-multiplied to the IC and Final Demand matrices.
- Existence of Negative Entries.
- Unrealistic assumption. Only in a few cases will firms supply all their products in the same proportions to their users.

Models of Transformation

- Model D [Fixed Product Sales Structure Assumption]
  - Each product has its own specific sales structure, irrespective of the industry where it is produced.
  - Does not require the use of a square SUT.

  - Application:
    - Compute for Matrix D [market share matrix] to derive the transformation matrix.
    - **Post-multiply** the Domestic Use Table to the Transformation Matrix to obtain the Industry-by-Industry IOT.

Model D

Supply Table at Basic Prices

	AGR	MFG	SER	Domestic Output
AGR	26	5	7	38
MFG	1	403	40	444
SER	13	26	717	756
Total Output at BP	**40**	**433**	**765**	**1238**

D Matrix: Market Share Matrix

	AGR	MFG	SER
AGR	0.68	0.00	0.02
MFG	0.14	0.91	0.03
SER	0.19	0.09	0.95

For the first row, we have 26/38, 5/38 and 7/38

- Transformation Matrix = **D**

Transformation Matrix for Model D

	AGR	MFG	SER
AGR	0.68	0.00	0.02
MFG	0.14	0.91	0.03
SER	0.19	0.09	0.95

Model D

Industry-by-Industry IO Table (Model D)

	AGR	MFG	SER	Total IC	FCE	GFCF	Exports	Total FD	Output
AGR	3	8	4	15	15	1	10	25	**40**
MFG	8	101	49	158	43	68	165	275	**433**
SER	5	71	205	281	332	26	125	484	**765**
Imports	3	124	78	205	75	37	245	357	**561**
GVA	22	130	428	580					**580**
Input	**40**	**433**	**765**	**1,238**	**465**	**131**	**544**	**1,141**	

- Entries in the Imports and GVA Vectors are unaltered from the Domestic Use Table.
- No negatives in the model.

Choice of IO Model

- ESA 1995 prefers the use of a product-by-product table in compiling IOTs (Eurostat, 2008 p.342).
- The choice of the model to use can also be influenced by the type of analysis an individual wishes to perform.
  - Product-by-Product IOT: well suited for the analysis of homogenous production units such as productivity, energy policy, cost structure comparison.
  - Industry-by-Industry IOT: well suited for industry analyses such as tax reform, impact analysis, fiscal policy, and monetary policy.
- In practice, NSOs compile their IOTs using Model A (product technology) and Model D (fixed product sales structure).
- Other issues are also considered in choosing what transformation model to use.
  - availability of resources, relevance, appropriateness of source data, statistical policy, international obligations, history and traditions

Sources and Solutions to Negative Entries

- Sources of Negatives
  - Product Technology Assumption is Incorrect
  - Heterogeneity in data classifications
  - Error in Data
  - Negatives in the Transformation Matrices (de Mesnard, 2011)
- Solutions to Negatives
  - Manually remove negatives then RAS [commonly used in practice]
  - Mathematical Optimization Procedures (see ten Raa and Rueda-Cantuche, 2013)

SUT-IOT Transformation using Georgia data

Structure of Input-Output Tables

Introduction

Fundamental Objective: to analyse the interdependence of industries in an economy

Basic Methodology:

- An input-output model consists of a system of linear equations, each describing the distribution of an industry's output throughout the economy
- using observed economic data for a specific geographic region and particular time period

Notation and Fundamental Relationships

Essential set of data for an input-output model:

- z_{ij} representing the monetary value of transaction from industry i to j
- f_i representing final demand for sector i's output

<u>Assume</u>: the economy can be categorized into n sectors, such that:

$$x_i = z_{i1} + z_{i2} + \cdots + z_{in} + f_i$$

$$x_i = \sum_{j=1}^{n} z_{ij} + f_i$$

Notation and Fundamental Relationships

There will be an equation like this that identifies sales of the output of each of the n sectors, thus we have this system of n equations:

$$x_1 = z_{11} + z_{12} + \cdots + z_{1n} + f_1$$
$$x_2 = z_{21} + z_{22} + \cdots + z_{2n} + f_2$$
$$\vdots$$
$$x_n = z_{n1} + z_{n2} + \cdots + z_{nn} + f_n$$

or:

$$x_i = z_{i1} + z_{i2} + \cdots + z_{in} + f_i \text{ , } i = 1,2, \ldots , n$$

Notation and Fundamental Relationships

$x_i = z_{i1} + z_{i2} + \cdots + z_{in} + f_i$, i = 1,2, … , n

This can be represented in matrix notation as:

$$\mathbf{x} = \mathbf{Zi} + \mathbf{f}$$

where:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}; \quad \mathbf{Z} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{nn} \end{bmatrix}$$

$$\mathbf{i} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}; \quad \mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{pmatrix}$$

Input-Output Table

		PRODUCERS AS CONSUMERS								FINAL DEMAND				TOTAL OUTPUT
		Agri	Mining	Const.	Manuf.	Trade	Transp.	…	Other	Personal Consumption Expenditures	Gross Private Domestic Investment	Govt. Purchases of Goods & Services	Net Exports of Goods & Services	
PRODUCERS	Agriculture													
	Mining													
	Construction													
	Manufacturing													
	Trade													
	Transportation													
	⋮													
	Other Industry													
Imports														
VALUE ADDED	Employees													
	Business Owners & Capital													
	Government													
TOTAL OUTPUT														

OUTPUTS

INPUTS

Z

f

x

m

v

x'

Technical Coefficients

$z_{ij} = f(x_j)$

The technical coefficients,

$a_{ij} = z_{ij} / x_j$

= inputs from sector i to sector j / total output of sector j

- These are assumed to be unchanging and viewed as measuring fixed relationships between a sector's output and its inputs
- Given fixed technical coefficients, the proportion of sector i's use of inputs from j and k is also fixed

Hence, $z_{ij} = a_{ij} x_j$

Technical Coefficients

As a system of equations:

$$x_i = z_{i1} + z_{i2} + \cdots + z_{in} + f_i$$

$$x_i = a_{i1}x_{i1} + a_{i2}x_{i2} + \cdots + a_{in}x_{in} + f_i \quad = \quad \sum_{j=1}^{n} a_{ij}\, x_{ij} + f_i$$

In matrix notation:

$$x = Zi + f$$

$$= Ax + f$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$$

$$\hat{\mathbf{x}} = \begin{bmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_n \end{bmatrix} \rightarrow \hat{\mathbf{x}}^{-1} = \begin{bmatrix} 1/x_1 & 0 & \dots & 0 \\ 0 & 1/x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1/x_n \end{bmatrix}$$

Technical Coefficients

		Agriculture	Manufacturing	Business Services	Total Final Demand	TOTAL OUTPUT
		c1	c2	c3		
Agriculture	c1	10,000	20,000	5,000	15,000	50,000
Manufacturing	c2	5,000	40,000	20,000	1,35,000	2,00,000
Business Services	c3	2,500	30,000	50,000	1,57,500	2,40,000
Imports	M	13,000	30,000	30,000	35,500	1,08,500
Intermediate input total		35,500	1,20,000	1,05,000	3,43,000	6,03,500
Value Added + TLS		19,500	80,000	1,45,000	7,000	2,51,500
TOTAL		50,000	2,00,000	2,50,000	3,50,000	**8,50,000**

Technical Coefficients Matrix →

$$\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$$

0.200	0.100	0.020
0.100	0.200	0.080
0.050	0.150	0.200

Leontief Inverse

Rearranging $\mathbf{x} = \mathbf{Ax} + \mathbf{f}$,

$$\mathbf{x} - \mathbf{Ax} = \mathbf{f}$$

$$(\mathbf{I} - \mathbf{A})\,\mathbf{x} = \mathbf{f}$$

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{f} = \mathbf{Lf}$$

Leontief Inverse (or the total requirements matrix):

$$(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{L}$$

As a system of equations,

$$x_i = a_{i1}x_{i1} + a_{i2}x_{i2} + \cdots + a_{in}x_{in} + f_i = \sum_{j=1}^{n} a_{ij}\,x_{ij} + f_i$$

$$x_i = L_{i1}f_{i1} + L_{i2}f_{i2} + \cdots + L_{in}f_{in} = \sum_{j=1}^{n} L_{ij}\,f_{ij}$$

Leontief Inverse

$$x_i = L_{i1}f_{i1} + L_{i2}f_{i2} + \cdots + L_{in}f_{in}$$

$$x = L\,f$$

- The model makes clear the dependence of the gross output on the values of each of the final demands.
- Thus, the Leontief inverse can be used to model the multiplier effects of a change in final demand on outputs.

References

For SUT to IOT

- Eurostat (2008). Eurostat Manual of Supply, Use and Input-Output Tables. Luxembourg.

For structure of Input-Output tables

- Miller, R. E., & Blair, P. D. (2009). Input-output analysis: foundations and extensions. 2nd Edition. Cambridge university press.

Thank you!